B.U.G. Newsletter

February 2015

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Is Love in the Air?

by Jennifer L. Brown

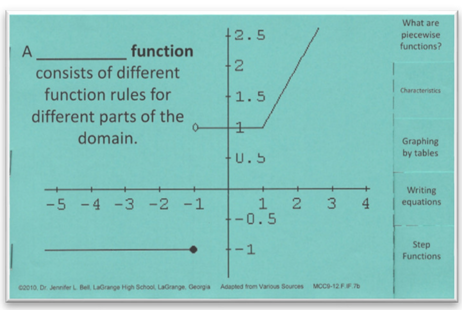
Are you students loving the engaging activities from the CRMC workshops? I certainly hope they are. If your semester is anything like mine, it is in full swing, and the days seem to be non-stop. This month's newsletter includes the graphic organizers, foldable, and activities for piecewise functions and matrices from the Spring 2015 Follow-up Workshop for those teachers who were unable to attend. Honestly, my students struggled with piecewise functions. I think a lot of the issues were related to the concept of domain, which is the rationale for some of the enclosed activities. Nearly all of these files are on my website in either Word or PowerPoint files so you can edit as needed. If you do not like to use foldables with your students, I included most of the material in graphic organizer form. The other information can be copied and pasted from the PowerPoint files if needed. If you have any questions, please let me know.

Dr. Brown ☺

FOR MORE IDEAS AND ACTIVITIES

Bell's Useful Guide for Teachers

www.bugforteachers.com/crmc.html



Directions for making the piecewise function foldable (pictured above)

1. Print two sets of the original file.
2. Order the pages 1, 1 (rotated 180 degrees counter clockwise), 2, 2 (rotated 180 degrees counter clockwise), 3, 3 (rotated 180 degrees counter clockwise), 4, 4 (rotated 180 degrees counter clockwise), 5, 5 (rotated 180 degrees counter clockwise), 6, 6 (rotated 180 degrees counter clockwise), 7, 7 (rotated 180 degrees counter clockwise).
3. Copy the set front and back.
4. Cut the papers in half. You should have two foldables per stack.
5. Remove the dotted rectangles to create the tabs.
6. Staple the half sheets on the left side.

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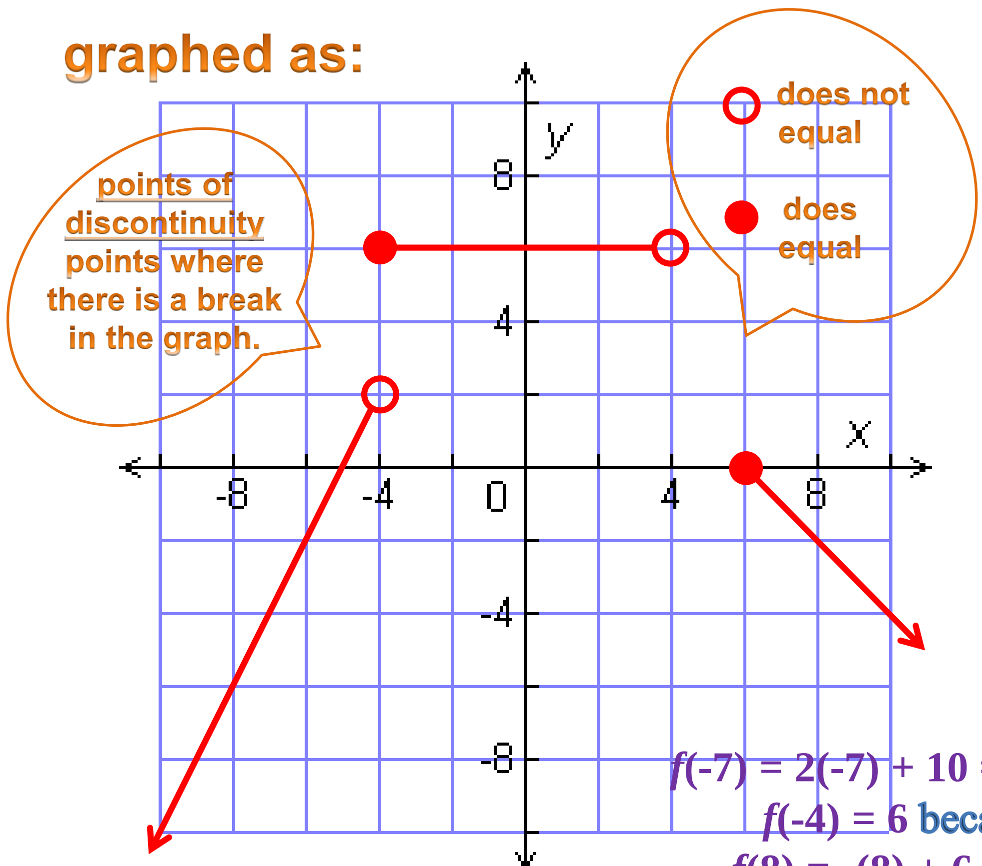
Piecewise Functions

A **piecewise function** consists of different function rules for different parts of the domain.

graphed as:



written as:

$$f(x) = \begin{cases} 2x + 10 & (x < -4) \\ 6 & (-4 \leq x < 4) \\ -x + 6 & (x \geq 6) \end{cases}$$

OR

$$f(x) = \begin{cases} 2x + 10 & (-\infty, -4) \\ 6 & [-4, 4) \\ -x + 6 & [6, \infty) \end{cases}$$

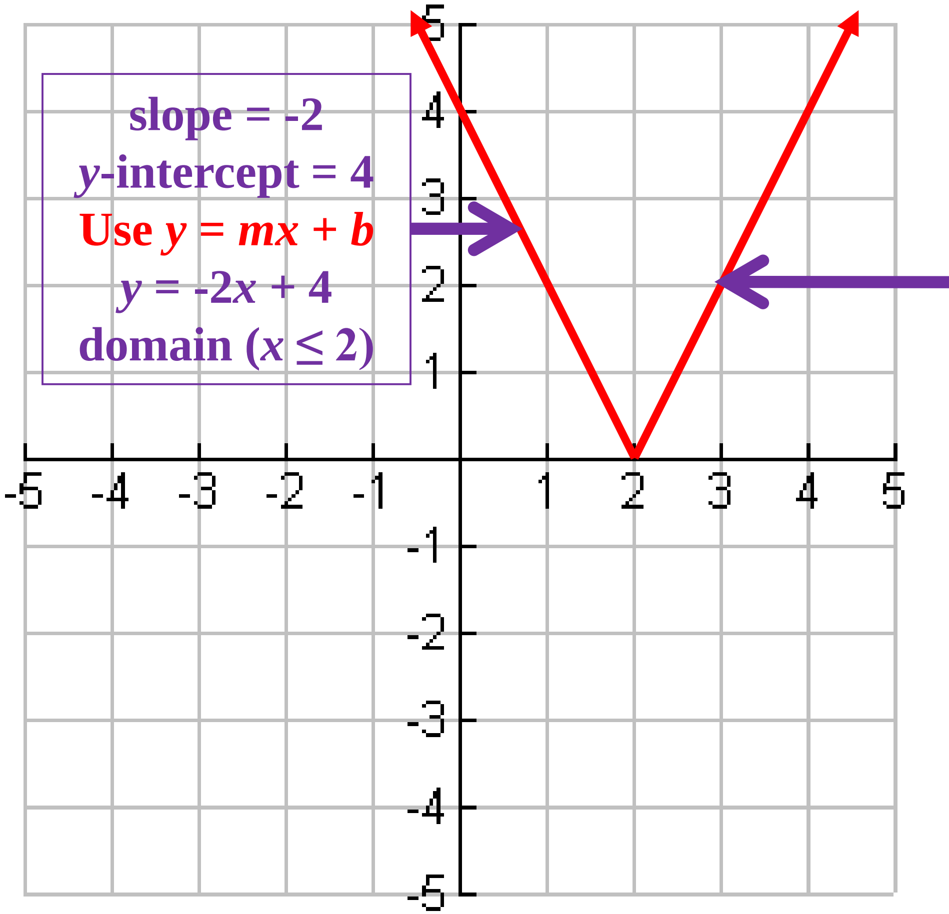
indicates the restricted domain

$f(-7) = 2(-7) + 10 = -4$ because $x = -7$ exists in the first equation only.

$f(-4) = 6$ because $x = -4$ exists in the second equation only.

$f(8) = -(8) + 6 = -2$ because $x = 8$ exists in third equation only.

How to write absolute value functions as piecewise functions



slope = 2 (*opposite of the left side)

y-intercept = unknown

Pick a point (3, 2)

& use $y_2 - y_1 = m(x_2 - x_1)$

$y - 2 = 2(x - 3)$

$y - 2 = 2x - 6$

$+2 \qquad +2$

$y = 2x - 4$

domain ($x > 2$)

Written as an absolute value function:

$$f(x) = 2\,|\,x - 2\,|$$

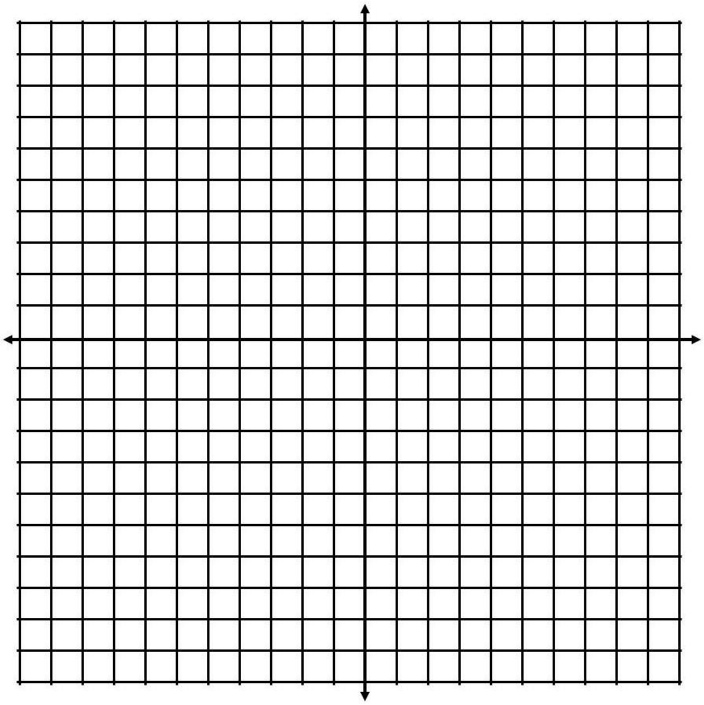
Written as a piecewise function:

$$f(x) = \begin{cases} -2x + 4 & (x \le 2) \\ 2x - 4 & (x > 2) \end{cases}$$

What are piecewise functions?

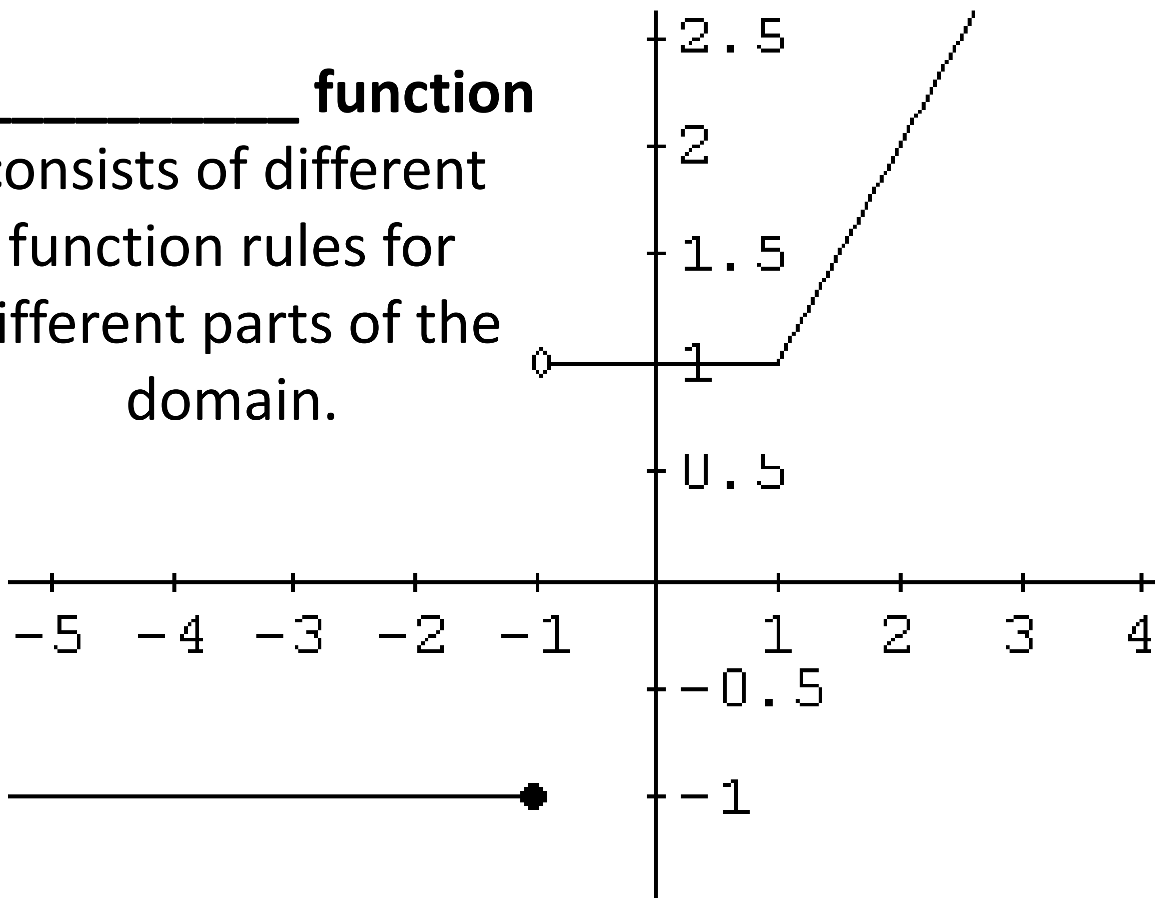
1. Graph $f(x) = x^2 + 1$.
2. Erase part of the graph.

- What is the domain?
- What is the NEW equation?



What are piecewise functions?

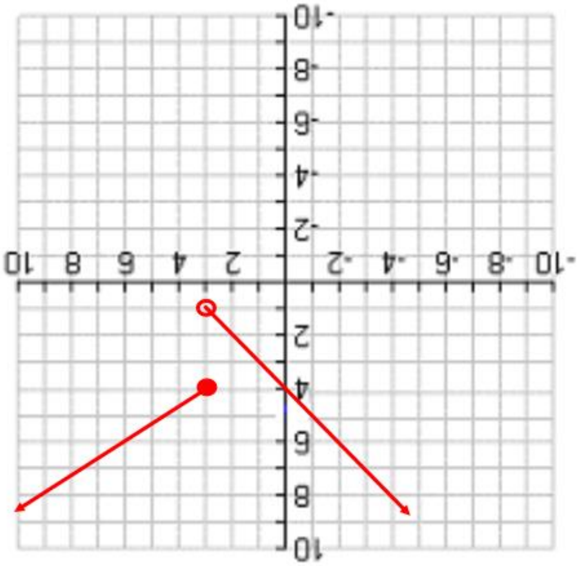
A ___________ **function** consists of different function rules for different parts of the domain.



Characteristics

7. What is the domain for the first (left) ray? The Range?

8. What is the domain for the second (right) ray? The Range?



What do these symbols mean?

● ____________ ○ ____________

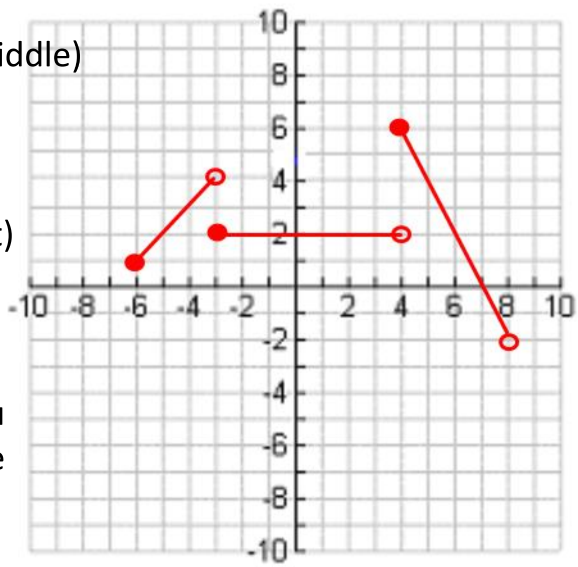
3. What is the domain for the first (left) segment? The Range?

Characteristics

4. What is the domain for the second (middle) segment? The Range?

5. What is the domain for the third (right) segment? The Range?

6. How many equations do you think you would have to use to write rule for the following piecewise function?



Both of the following notations can be used to describe a piecewise function over the function's domain:

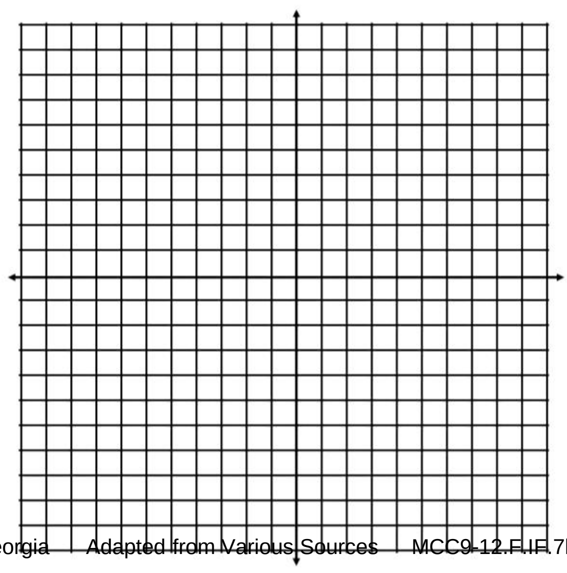
$$f(x)=\begin{cases}2x & if[-5,2) \\ 5 & if[2,6]\end{cases} \quad \textbf{OR} \quad f(x)=\begin{cases}2x, -5\le x<2 \\ 5, \quad 2\le x\le 6\end{cases}$$

9. Complete the following table of values for the piecewise function.

10. Graph the ordered pairs from your table to graph the piecewise function

Graphing by tables

x	$f(x)$
-5	
-3	
0	
1	
1.7	
1.9	
2	
2.2	
4	
6	



Graphing by tables

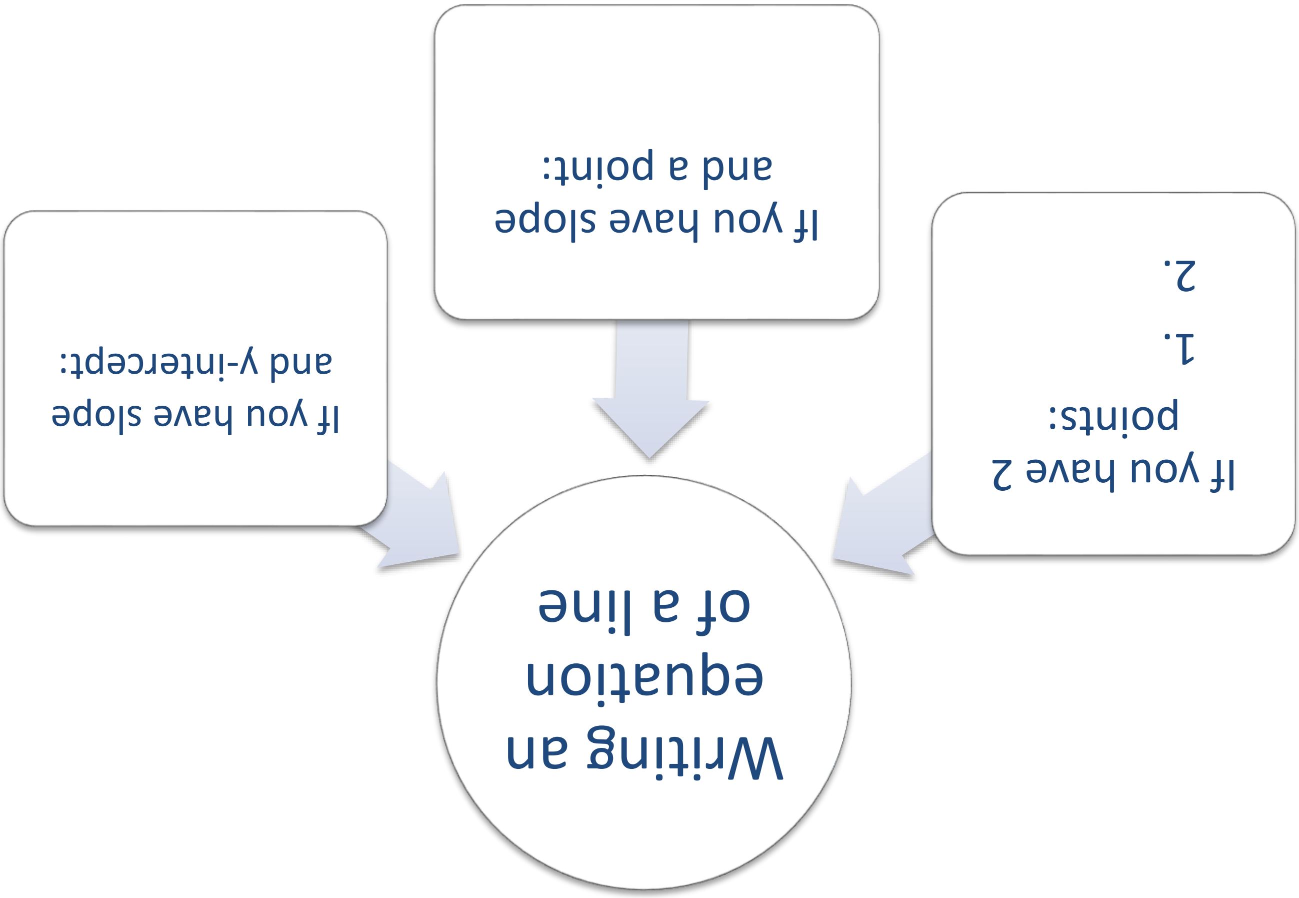
11. How many pieces does your graph have? Why?

12. Are the pieces rays or segments? Why?

13. Are all the endpoints solid dots or open dots or some of each? Why?

14. Were all these x values necessary to graph this piecewise function? Could this function have been graphed using less points?

15. Which x values were "critical" to include in order to sketch the graph of this piecewise function?



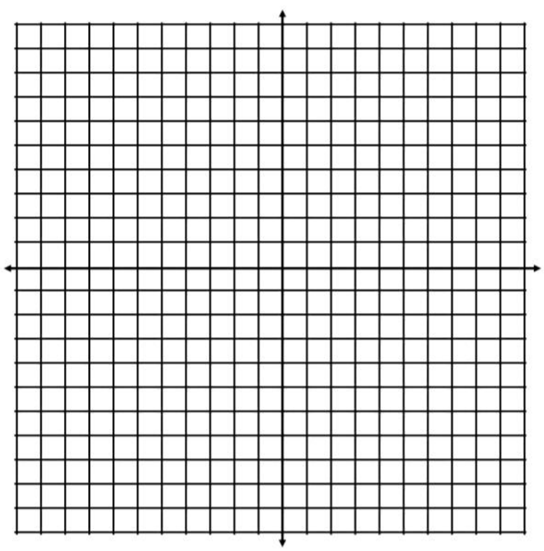
Writing
equations
of lines

16. Graph this piecewise function by completing a table of values:

$$f(x) = \begin{cases} x+3, & -8 \le x < 1 \\ 10-2x, & 1 \le x \le 7 \end{cases}$$

x	$f(x)$

x	$f(x)$



17. How many pieces does your graph have? Why?

18. Are the pieces rays or segments? Why?

19. Are all the endpoints filled circles or open circles or some of each? Why?

Graphing by tables

20. Was it necessary to evaluate both pieces of the function for the x-value 1? Why or why not?

21. Which x values were "critical" to include in order to graph this piecewise function? Explain.

22. Write the equations of the lines that contain each segment.

a) Left segment equation

a) Middle equation

b) Right equation

23. List the domain of each segment.

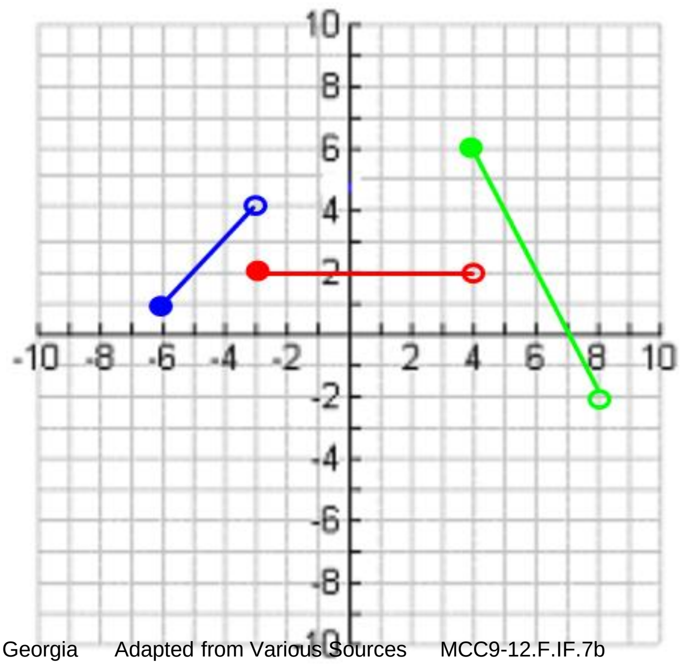
a) Left segment equation

a) Middle equation

b) Right equation

24. Put the domain together with the equations to write the equation for the piecewise function.

$$f(x) = \begin{cases} \\ \\ \end{cases}$$



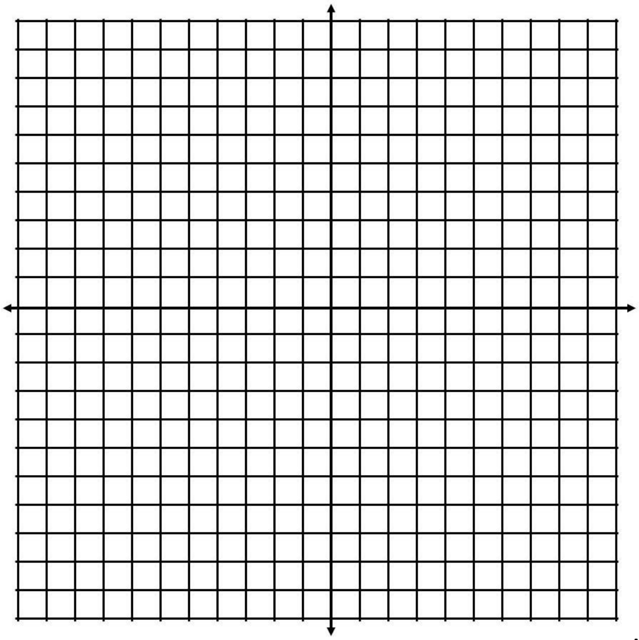
Writing equations

25. Graph the absolute value function. $f(x) = -|x + 2|$

26. List all transformations compared to the parent function.

27. Write the absolute value function as a piecewise function.

x	$f(x)$



Writing equations

Step Functions

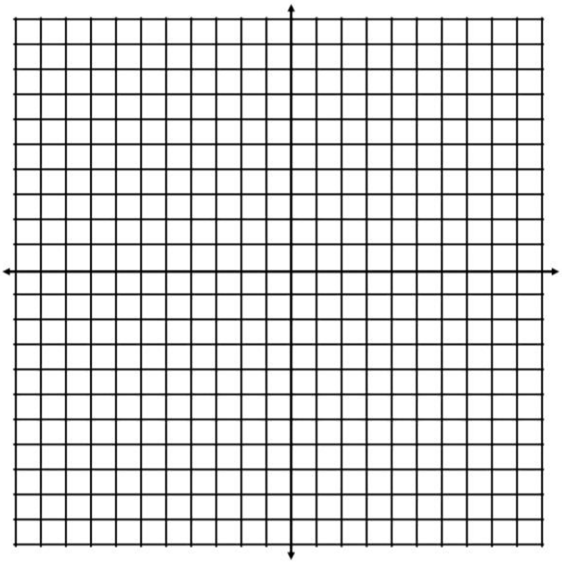
28. Graph the step function.

29. What is the domain?

30. What is the range?

31. Why is the range not all real numbers?

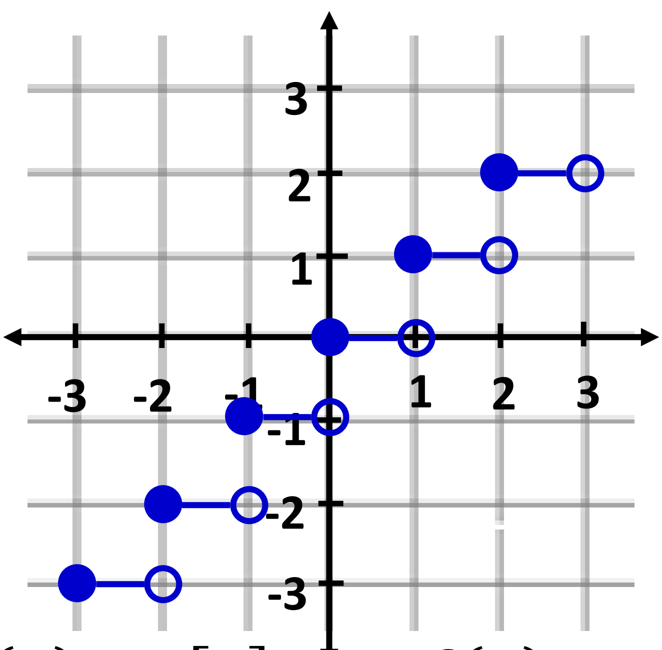
$$f(x)=\begin{cases}1, & 0\le x<1\\ 3, & 1\le x<3\\ 5, & 3\le x<5\\ 7, & 5\le x<7\end{cases}$$



A __________ **function** is a piecewise function that consists of different constant range values for different intervals of the function’s domain.

Greatest integer function

Rounding-down function



$f(x) = [x]$ ***OR*** $f(x) = \lfloor x \rfloor$

Step Functions

 Adapted from Various Sources MCC9-12.F.IF.7b

PRACTICE Use the following piecewise functions to answer the following questions:

a) What is the domain? Range?

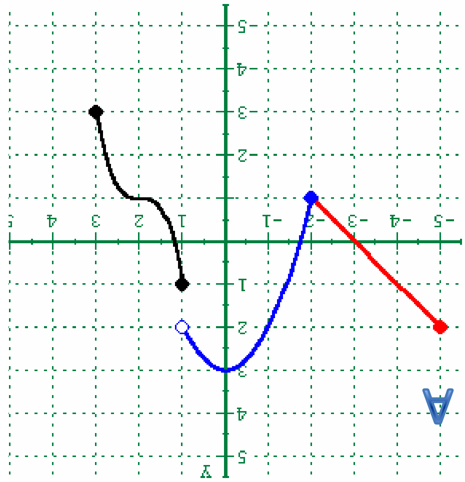
b) What are the x-intercepts? y-intercepts?

c) Describe the end behavior.

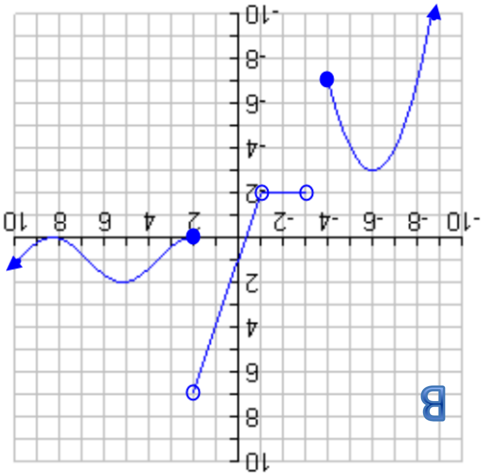
d) Where are the intervals of increasing? Decreasing? Remaining constant?

e) Find *f*(2).

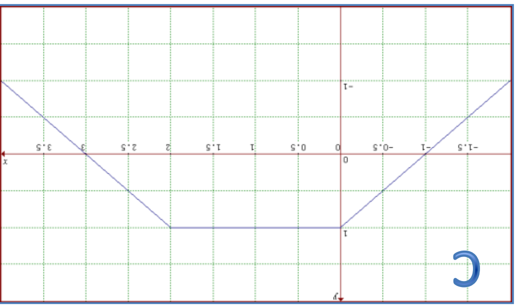
A



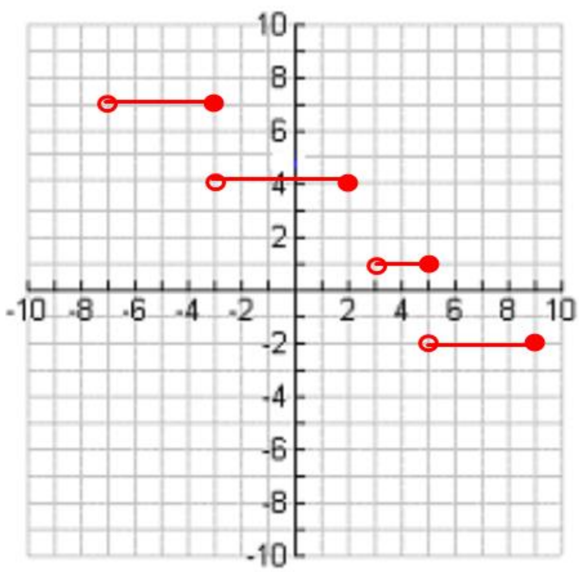
B



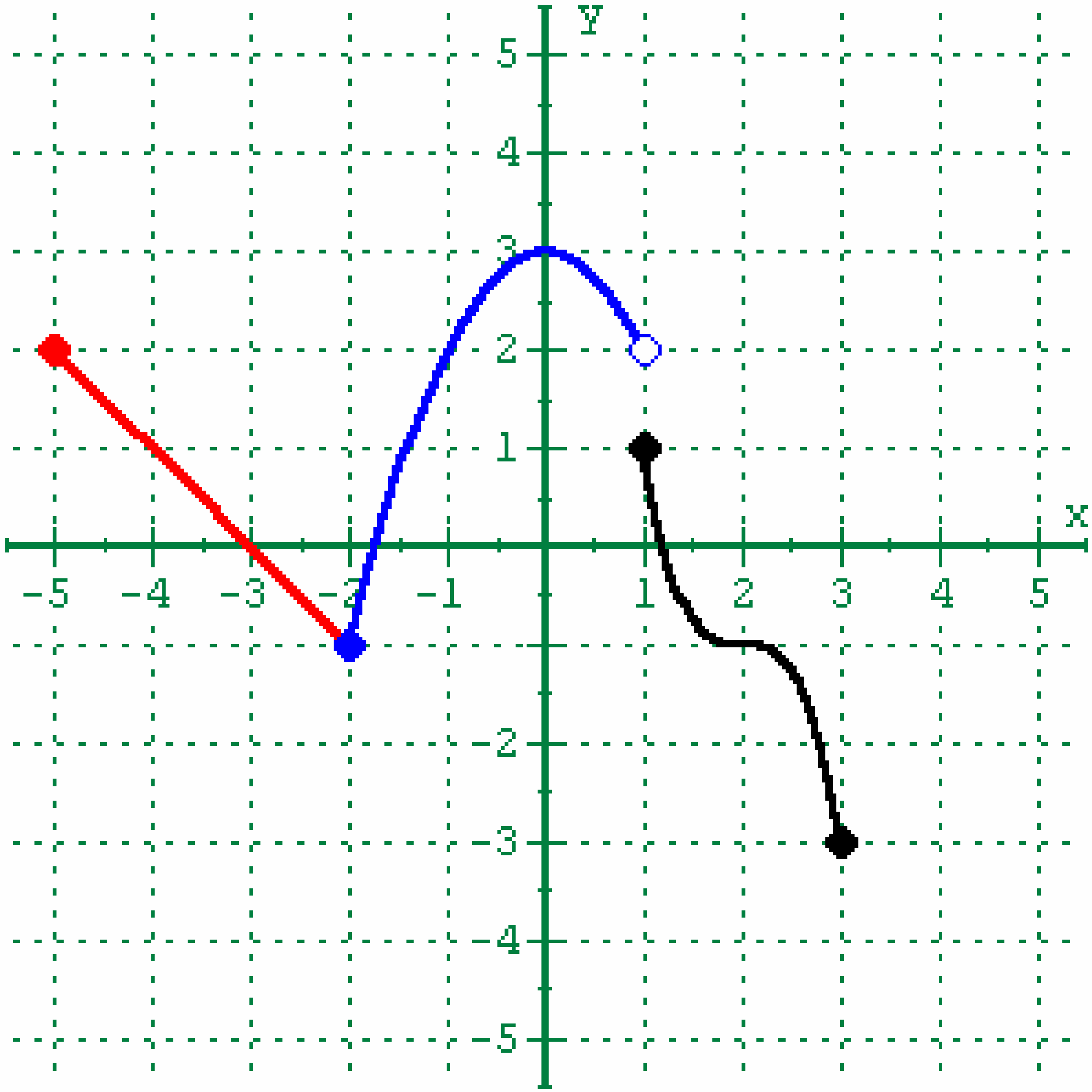
C



32. Write the equation of the piecewise function that matches the step function graph.



Introduction to Piecewise Functions

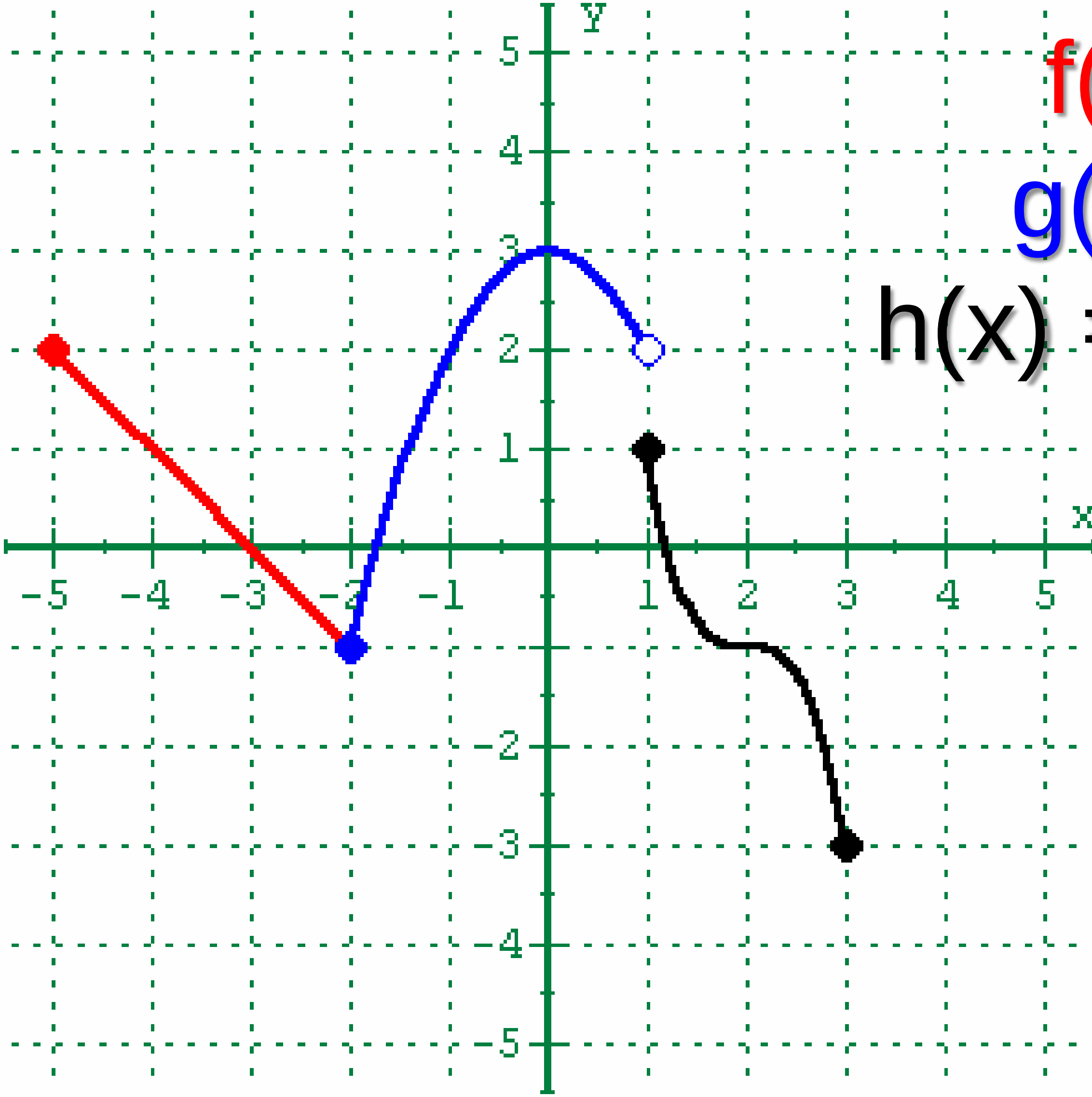


1. Pick a number between -5 and 3.
2. Determine which "piece" of the function corresponds to your domain number.
3. Write the equation for the function.
4. Find your fellow function-mates.

$f(x) = -x - 3$

$g(x) = -x^2 + 3$

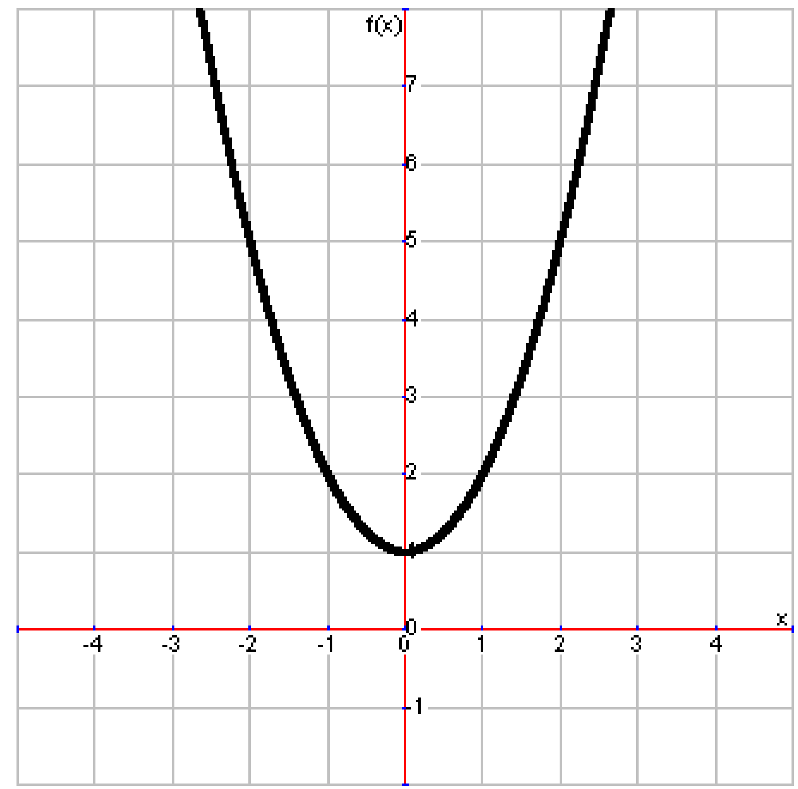
$h(x) = -2(x - 2)^3 - 1$



Piecewise Functions: Activity 1

Make a table and graph for the following the function.

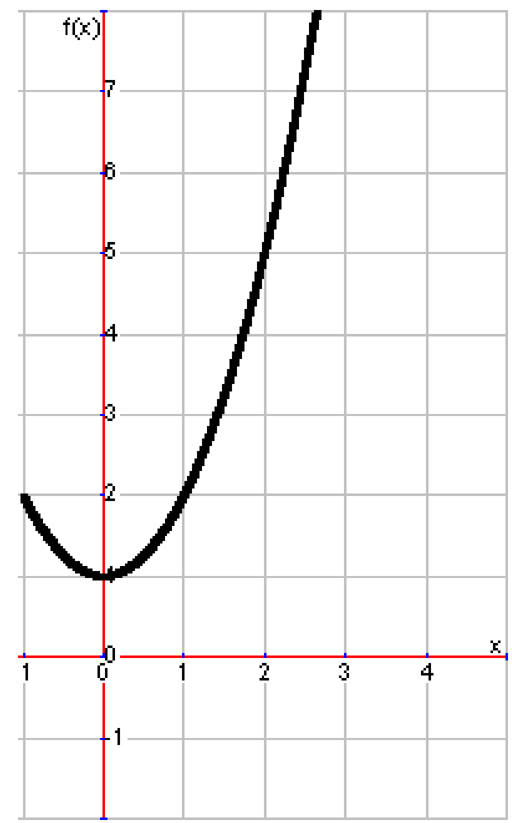
$$f(x) = x^2 + 1$$



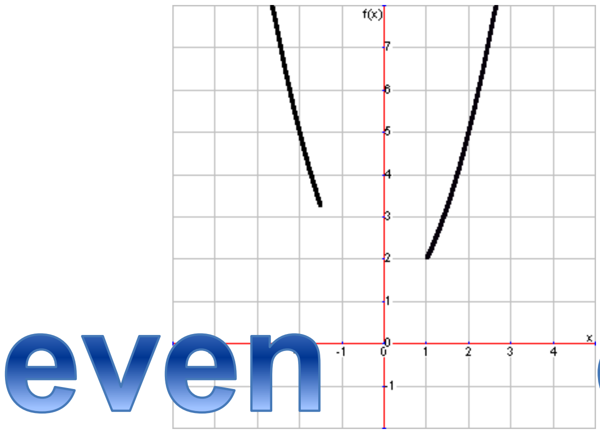
Take away "part" of the graph by either erasing or folding the paper.

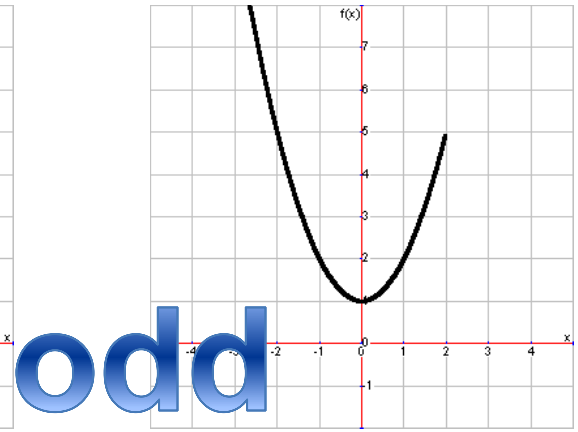
Domain?

Equation?



Discuss restricted domains.





- Write your function with this restricted domain.
- With a neighbor, check each other's work.

Piecewise Functions: Activity 2

Piecewise Function Activity

- Graph the following equations with the restricted domain:

EVEN -

$$f(x) = -2x + 3, x \le 2$$

ODD -

$$f(x) = (x-2)^2, x > 2$$

Piecewise Functions: Activity 3

Graph:

$$c(x) = \begin{cases} 1 + 1.2\sqrt{x-1}, & x \le 3.5 \\ 4 - 0.5(x-5)^2, & x \ge 3.5 \text{ and } \le 6.5 \\ 1 + 1.2\sqrt{-(x-9)}, & x \ge 6.5 \end{cases}$$

$$d(x) = 1, \quad x \ge 1 \text{ and } x \le 9$$

$$e(x) = 1 - \sqrt{1 - (x-2.5)^2}$$

$$g(x) = 1 - \sqrt{1 - (x-7.5)^2}$$

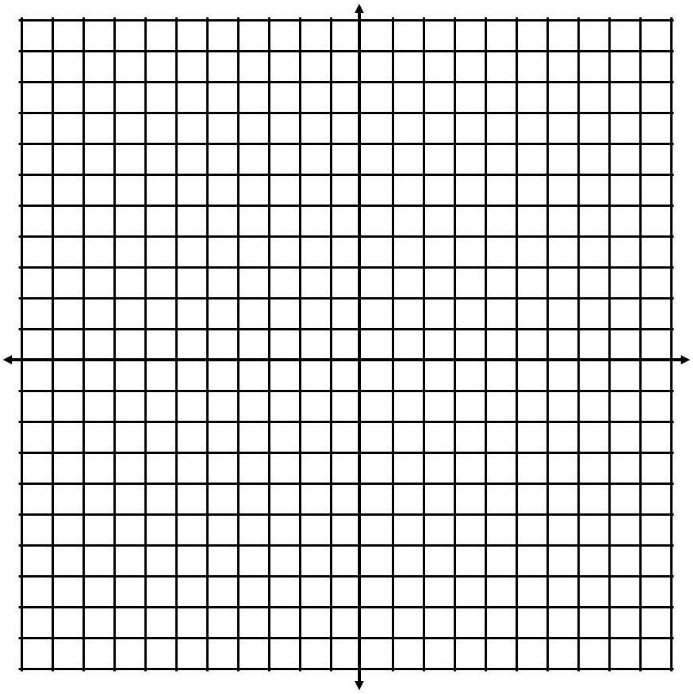
$$h(x) = 2 + |x - 5.5|, \quad x \ge 5.2 \text{ and } x \le 5.8$$

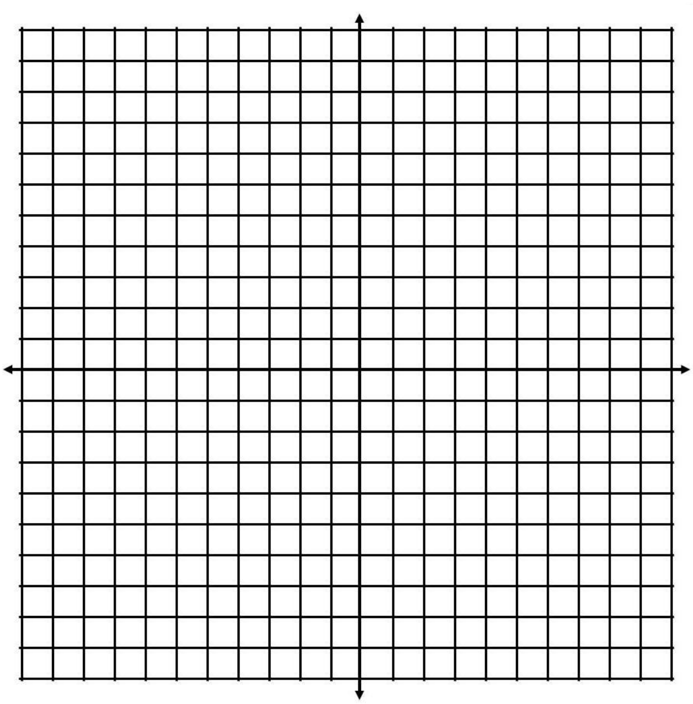
- Which function represents each part of the car?
- Change the piecewise functions to increase the size of the car.

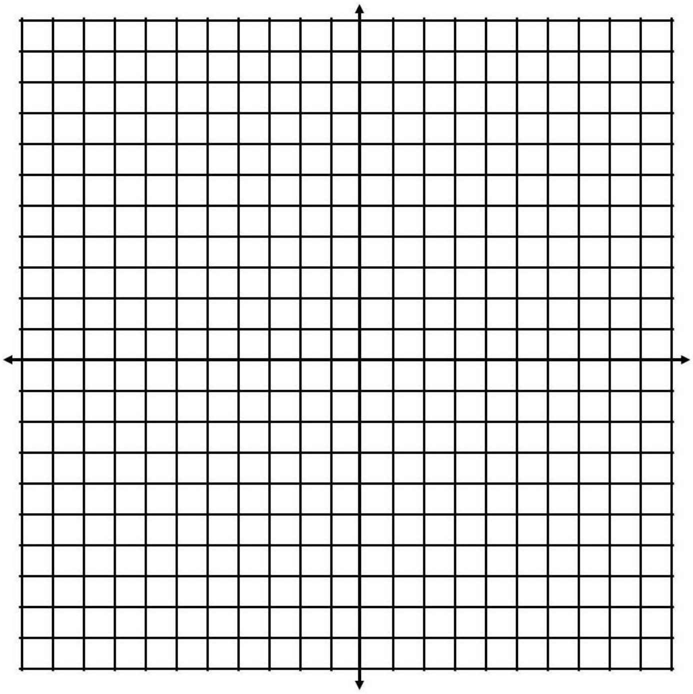
Piecewise Functions: Activity 4

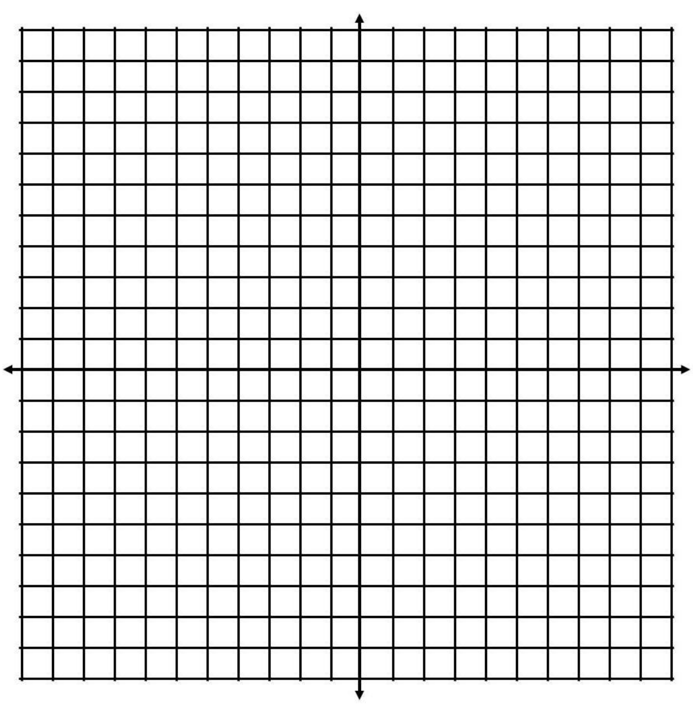
Piecewise Comic Strip

1. Draw a function given the restricted domain.
2. Write a narrative for the function.
3. Group with 3 others & create a comic strip.









Piecewise Choice Board

Directions:

1. Complete the center block.
2. Choose 2 more blocks, but your blocks must form a straight line which contains the center.

Use your piecewise function to write the equations.	Use your piecewise function write a letter to the editor.	Use your piecewise function to write a story about the function's path.
Use your piecewise function to illustrate a real-world situation.	Draw a piecewise function containing at least 3 pieces.	Use your piecewise function to find the slope of each piece.
Use your piecewise function to make a review game.	Use your piecewise function to create a storyboard.	Use your piecewise function to explain piecewise functions to a new student.

matrix

Dimensions (m X n)
the number of rows (m) and the number of columns (n) in the matrix (e.g., 3 X 3).

rectangular array of numbers
(Plural: Matrices)

Column 1

Row 1

$$\begin{bmatrix} -7 & 0 & 13 \\ -2 & 4 & -2 \\ 12 & 1 & 10 \end{bmatrix}$$

Entry (or element) –
Numbers inside a matrix

Location (or "address") –
Number of the row and column where the entry is located (e.g., 3, 1).

How to Add and Subtract Matrices

**It is only possible to add or subtract two matrices, if they have the same dimensions.

To find the sum, add corresponding entries
(or numbers in the same location.)

$$\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} + \begin{bmatrix} g & h & i \\ j & k & l \end{bmatrix} = \begin{bmatrix} a+g & b+h & c+i \\ d+j & e+k & f+l \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 7 \\ -4 & 5 & 1 \end{bmatrix} + \begin{bmatrix} 4 & -8 & 1 \\ 9 & 5 & 0 \end{bmatrix} = \begin{bmatrix} 2+(4) & 0+(-8) & 7+(1) \\ -4+(9) & 5+(5) & 1+(0) \end{bmatrix} = \begin{bmatrix} 6 & -8 & 8 \\ 5 & 10 & 1 \end{bmatrix}$$

To find the difference, subtract corresponding entries.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a-e & b-f \\ c-g & d-h \end{bmatrix}$$

$$\begin{bmatrix} 5 & -3 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} -4 & 7 \\ 9 & 2 \end{bmatrix} = \begin{bmatrix} 5-(-4) & -3-(7) \\ 1-(9) & 0-(2) \end{bmatrix} = \begin{bmatrix} 9 & -10 \\ -8 & -2 \end{bmatrix}$$

Matrix Multiplication

- The **inner dimensions** must be the same (or the number of columns in the first matrix is equal to the number of rows in the second matrix).
- The **outer dimensions** become the dimensions of the resulting matrix (or the number of rows in the first matrix and the number of columns in the second matrix).

inner dimensions

2 x 3   3 x 2

outer dimensions

product dimensions

= 2 x 2

Steps for Matrix Multiplication

1. Multiply the corresponding positions of the first row and the first column.
2. Add the products.
3. Place answer in the first row, first column address.

$$\begin{bmatrix} 1 & -2 \\ 5 & -4 \end{bmatrix} \bullet \begin{bmatrix} 3 & -6 \\ 7 & -1 \end{bmatrix} = \begin{bmatrix} (1)(3)+(-2)(7) & \\ & \end{bmatrix}$$

4. Repeat steps 1 & 2 with the first row and second column.
5. Place answer in the first row, second column address.

$$\begin{bmatrix} 1 & -2 \\ 5 & -4 \end{bmatrix} \bullet \begin{bmatrix} 3 & -6 \\ 7 & -1 \end{bmatrix} = \begin{bmatrix} -11 & (1)(-6)+(-2)(-1) \\ & \end{bmatrix}$$

6. Repeat steps 1 & 2 with the second row and first column.
7. Place answer in the second row, first column address.

$$\begin{bmatrix} 1 & -2 \\ 5 & -4 \end{bmatrix} \bullet \begin{bmatrix} 3 & -6 \\ 7 & -1 \end{bmatrix} = \begin{bmatrix} -11 & -4 \\ (5)(3)+(-4)(7) & \end{bmatrix}$$

8. Repeat steps 1 & 2 with the second row and second column.
9. Place answer in the second row, second column address.

$$\begin{bmatrix} 1 & -2 \\ 5 & -4 \end{bmatrix} \bullet \begin{bmatrix} 3 & -6 \\ 7 & -1 \end{bmatrix} = \begin{bmatrix} -11 & -4 \\ -13 & (5)(-6)+(-4)(-1) \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 \\ 5 & -4 \end{bmatrix} \bullet \begin{bmatrix} 3 & -6 \\ 7 & -1 \end{bmatrix} = \begin{bmatrix} -11 & -4 \\ -13 & -26 \end{bmatrix}$$

Scalar Multiplication

A **scalar** is a real number (or 1 X 1 matrix).

Multiply the scalar with every entry in the matrix.

Every little kid gets a piece of candy! ☺

$$3 \cdot \begin{bmatrix} 2 & 0 & 7 \\ -4 & 5 & 1 \end{bmatrix} = \begin{bmatrix} 2(3) & 0(3) & 7(3) \\ -4(3) & 5(3) & 1(3) \end{bmatrix} = \begin{bmatrix} 6 & 0 & 21 \\ -12 & 15 & 3 \end{bmatrix}$$

Determinant

denoted as:

$$\det(A) = |A|$$

Special number associated with a square matrix.

play an important role in:

- finding the inverse of a matrix.
- solving systems of linear equations.

If the $\det(A) = 0$, the matrix is called a **singular matrix**.

Singular matrices do not have an inverse because you must multiply by $^{1}/_{\det(A)}$ to find the inverse. (You cannot ÷ 0.)

Determinant

Special number associated with a square matrix.

2 X 2 matrix

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - cb$$

$$\begin{bmatrix} 4 & -3 \\ 5 & 2 \end{bmatrix} = \begin{vmatrix} 4 & -3 \\ 5 & 2 \end{vmatrix} = 4(2) - 5(-3)$$

$$8 - (-15) = 23$$

3 X 3 matrix

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} \begin{matrix} a & b \\ d & e \\ g & h \end{matrix}$$

$$(aei + bfg + cdh) - (gec + hfa + idb)$$

$$\begin{bmatrix} 1 & 1 & 2 \\ -1 & 3 & 4 \\ 0 & 2 & 6 \end{bmatrix} = \begin{vmatrix} 1 & 1 & 2 \\ -1 & 3 & 4 \\ 0 & 2 & 6 \end{vmatrix} \begin{matrix} 1 & 1 \\ -1 & 3 \\ 0 & 2 \end{matrix}$$

$$(1)(3)(6) + (1)(4)(0) + (2)(-1)(2) - ((0)(3)(2) + (2)(4)(1) + (6)(-1)(1))$$

$$14 - 2 = 12$$

How to find the determinant for a 2 X 2 matrix

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} \rightarrow a \cdot d - c \cdot b$$

1. Multiply top left and bottom right.
2. Subtract.
3. Multiply top right and bottom left.
4. Simplify.

Example

$$\begin{vmatrix} 8 & 4 \\ 6 & 5 \end{vmatrix} \rightarrow 8 \cdot 5 - 6 \cdot 4 = 16$$

How to find the determinant for a 3 X 3 matrix

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \left| \begin{array}{ccc|cc} a & b & c & a & b \\ d & e & f & d & e \\ g & h & i & g & h \end{array} \right.$$

1. Copy the first and second columns & write them on the right side.

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \left| \begin{array}{ccc|cc} a & b & c & a & b \\ d & e & f & d & e \\ g & h & i & g & h \end{array} \right.$$

$$(aei + bfg + cdh)$$

2. Multiply & add the elements when the first row diagonals.

3. Subtract.

$$(aei + bfg + cdh) - (gec + hfa + idb)$$

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \left| \begin{array}{ccc|cc} a & b & c & a & b \\ d & e & f & d & e \\ g & h & i & g & h \end{array} \right.$$

3. Multiply & add the elements when the last row diagonals.

4. Simplify.

Example

$$\begin{bmatrix}1 & 1 & 2\\-1 & 3 & 4\\0 & 2 & 6\end{bmatrix} = \left|\begin{matrix}1 & 1 & 2\\-1 & 3 & 4\\0 & 2 & 6\end{matrix}\right|\begin{matrix}1 & 1\\-1 & 3\\0 & 2\end{matrix}$$

1. Copy the first and second columns & write them on the right side.

$$\begin{bmatrix}1 & 1 & 2\\-1 & 3 & 4\\0 & 2 & 6\end{bmatrix} = \left|\begin{matrix}1 & 1 & 2\\-1 & 3 & 4\\0 & 2 & 6\end{matrix}\right|\begin{matrix}1 & 1\\-1 & 3\\0 & 2\end{matrix}$$

$$(1)(3)(6)+(1)(4)(0)+(2)(-1)(2)$$

2. Multiply & add the elements when the first row diagonals.

3. Subtract.

$$\begin{bmatrix}1 & 1 & 2\\-1 & 3 & 4\\0 & 2 & 6\end{bmatrix} = \left|\begin{matrix}1 & 1 & 2\\-1 & 3 & 4\\0 & 2 & 6\end{matrix}\right|\begin{matrix}1 & 1\\-1 & 3\\0 & 2\end{matrix}$$

$$(1)(3)(6)+(1)(4)(0)+(2)(-1)(2)-\left((0)(3)(2)+(2)(4)(1)+(6)(-1)(1)\right)$$

3. Multiply & add the elements when the last row diagonals.

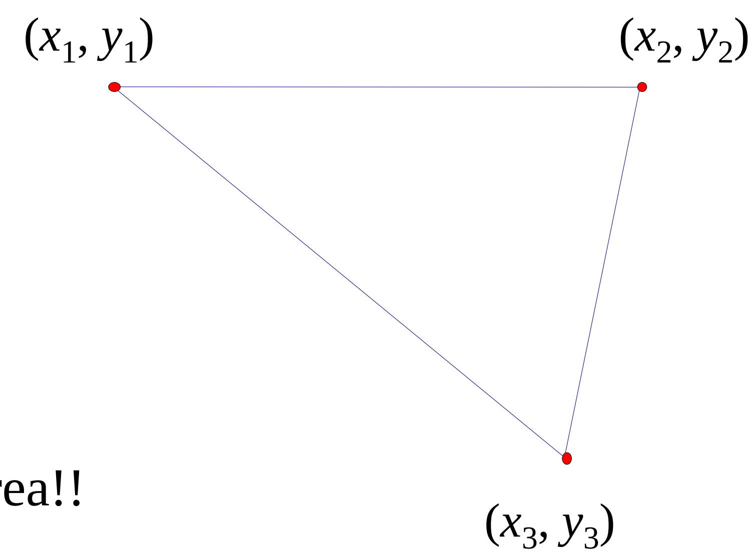
4. Simplify.

$$|A| = 14 - 2 = 12$$

Find the area of a triangle using the determinant

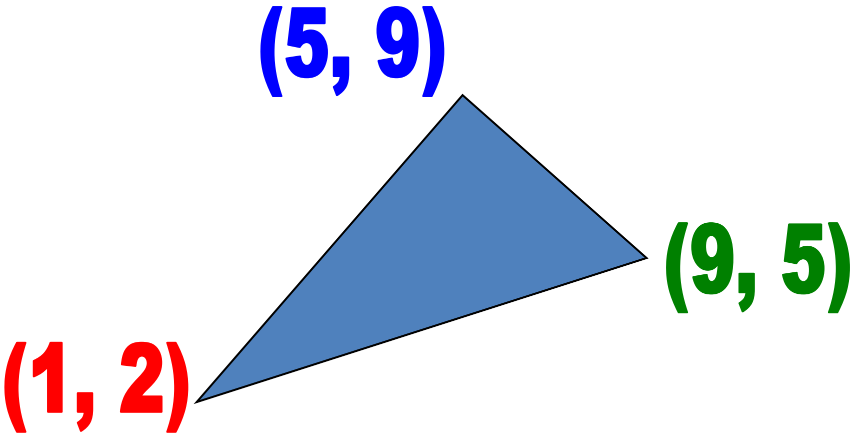
The area of a triangle with vertices (x_1,y_1), (x_2,y_2), and (x_3,y_3).

$$A = \pm\frac{1}{2}\begin{vmatrix} X1 & Y1 & 1 \\ X2 & Y2 & 1 \\ X3 & Y3 & 1 \end{vmatrix}$$



*Where ± is used to produce a positive area!!

Example



$$A = \pm\frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 5 & 9 & 1 \\ 9 & 5 & 1 \end{vmatrix}$$

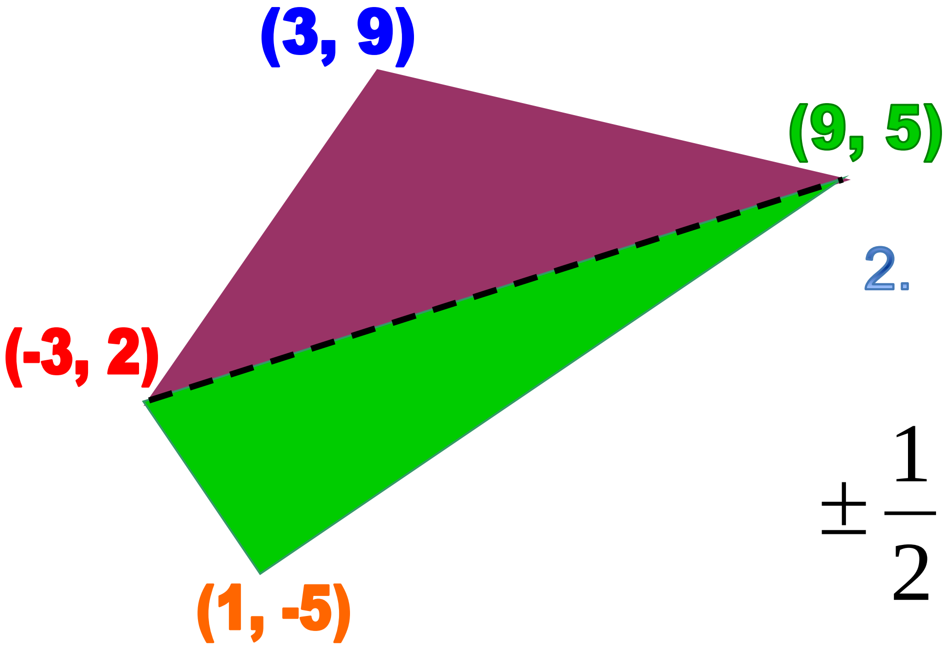
$$\pm\frac{1}{2}\left|\begin{matrix} 1 & 2 & 1 \\ 5 & 9 & 1 \\ 9 & 5 & 1 \end{matrix}\right|\begin{matrix} 1 & 2 \\ 5 & 9 \\ 9 & 5 \end{matrix}$$

$$\pm\tfrac{1}{2}\,(1\bullet 9\bullet 1 + 2\bullet 1\bullet 9 + 1\bullet 5\bullet 5 - (9\bullet 9\bullet 1 + 5\bullet 1\bullet 1 + 1\bullet 5\bullet 2)$$

$$\pm\tfrac{1}{2}\,(9 + 18 + 25 - 81 - 5 - 10)$$

$$\pm\tfrac{1}{2}(-44) = 22$$

How to find the area of a quadrilateral



1. Divide into 2 triangles.
2. Find the area of the top triangle.

$$\pm\frac{1}{2}\begin{vmatrix} -3 & 2 & 1 \\ 3 & 9 & 1 \\ 9 & 5 & 1 \end{vmatrix} = \pm\tfrac{1}{2}\left(-66\right) = 33$$

3. Find the area of the bottom triangle.

$$\pm\frac{1}{2}\begin{vmatrix} 9 & 5 & 1 \\ 1 & -5 & 1 \\ -3 & 2 & 1 \end{vmatrix} = \pm\tfrac{1}{2}\left(-96\right) = 48$$

4. Add the areas.

Purple Area = 33

Green Area = 48

Total Area = 81

How to solve a matrix equation

$$4\left(\begin{bmatrix} 3x & 1 \\ 0 & 6 \end{bmatrix} + \begin{bmatrix} 1 & -3 \\ 2 & -2y \end{bmatrix}\right) = \begin{bmatrix} -8 & -8 \\ 8 & 0 \end{bmatrix}$$

1. Add matrices inside parenthesis.

$$4\left(\begin{bmatrix} 3x+1 & -2 \\ 2 & 6-2y \end{bmatrix}\right) = \begin{bmatrix} -8 & -8 \\ 8 & 0 \end{bmatrix}$$

2. Distribute the scalar.

$$\begin{bmatrix} 12x+4 & -8 \\ 8 & 24-8y \end{bmatrix} = \begin{bmatrix} -8 & -8 \\ 8 & 0 \end{bmatrix}$$

3. Equate corresponding elements.

$$\begin{aligned} 12x + 4 &= -8 \\ -4 \quad & \quad -4 \\ \hline \frac{12x}{12} &= \frac{-12}{12} \\ x &= -1 \end{aligned}$$

$$\begin{aligned} 24 - 8y &= 0 \\ -24 \quad & \quad -24 \\ \hline \frac{-8y}{-8} &= \frac{-24}{-8} \\ y &= 3 \end{aligned}$$

4. Solve.

adapted from various sources

How to solve a linear system of equations using the determinant

**Equations MUST be in standard form!

2 variables

$$\begin{aligned} ax + by &= p \\ cx + dy &= q \end{aligned}$$

$$x = \frac{\text{Det}\left(\begin{bmatrix} p & b \\ q & d \end{bmatrix}\right)}{\text{Det}\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right)} \qquad y = \frac{\text{Det}\left(\begin{bmatrix} a & p \\ c & q \end{bmatrix}\right)}{\text{Det}\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right)}$$

OR

$$x = \frac{pd - bq}{ad - bc} \qquad y = \frac{aq - cp}{ad - bc}$$

Example:

$$\begin{aligned} 3x - 2y &= 2 \\ 2x - y &= 2 \end{aligned}$$

$$x = \frac{\begin{vmatrix} 2 & -2 \\ 2 & -1 \end{vmatrix}}{\begin{vmatrix} 3 & -2 \\ 2 & -1 \end{vmatrix}} = \frac{(2)(-1)-(2)(-2)}{(3)(-1)-(-2)(2)} = 2$$

$$y = \frac{\begin{vmatrix} 3 & 2 \\ 2 & 2 \end{vmatrix}}{\begin{vmatrix} 3 & -2 \\ 2 & -1 \end{vmatrix}} = \frac{(3)(2)-(2)(2)}{(3)(-1)-(-2)(2)} = 2$$

(2,2)

3 variables

$$\begin{aligned} ax + by + cz &= p \\ dx + ey + fz &= q \\ gx + hy + iz &= r \end{aligned}$$

$$x = \frac{\text{Det}\left(\begin{bmatrix} p & b & c \\ q & e & f \\ r & h & i \end{bmatrix}\right)}{\text{Det}\left(\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\right)} \qquad y = \frac{\text{Det}\left(\begin{bmatrix} a & p & c \\ d & q & f \\ g & r & i \end{bmatrix}\right)}{\text{Det}\left(\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\right)} \qquad z = \frac{\text{Det}\left(\begin{bmatrix} a & b & p \\ d & e & q \\ g & h & r \end{bmatrix}\right)}{\text{Det}\left(\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\right)}$$

Example:

$$\begin{aligned} 3x + y + 2z &= 9 \\ -2x + 2y + 3z &= 6 \\ 2x - y + z &= 8 \end{aligned}$$

$$x = \frac{\begin{vmatrix} 9 & 1 & 2 \\ 6 & 2 & 3 \\ 8 & -1 & 1 \end{vmatrix}}{\begin{vmatrix} 3 & 1 & 2 \\ -2 & 2 & 3 \\ 2 & -1 & 1 \end{vmatrix}} = 1$$

$$y = \frac{\begin{vmatrix} 3 & 9 & 2 \\ -2 & 6 & 3 \\ 2 & 8 & 1 \end{vmatrix}}{\begin{vmatrix} 3 & 1 & 2 \\ -2 & 2 & 3 \\ 2 & -1 & 1 \end{vmatrix}} = -2$$

$$z = \frac{\begin{vmatrix} 3 & 1 & 9 \\ -2 & 2 & 6 \\ 2 & -1 & 8 \end{vmatrix}}{\begin{vmatrix} 3 & 1 & 2 \\ -2 & 2 & 3 \\ 2 & -1 & 1 \end{vmatrix}} = 4$$

(1,–2,4)

 MCC9-12.N.VM.12; MCC9-12.A.REI.8

HOW TO FIND THE INVERSE OF A 2 X2 MATRIX

$$A=\begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad A^{-1}=\frac{1}{|A|}\cdot\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}=\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Inverse → opposite

$$A=\begin{bmatrix} 5 & -1 \\ 4 & 3 \end{bmatrix} \qquad A^{-1}=\frac{1}{19}\cdot\begin{bmatrix} 3 & 1 \\ -4 & 5 \end{bmatrix}=\begin{bmatrix} \frac{3}{19} & \frac{1}{19} \\ \frac{-4}{19} & \frac{5}{19} \end{bmatrix}$$

1. Switch the locations for d and a.
2. Add a negative to the entries in b and c.
3. Multiply the matrix times $^{1}/_{\det(A)}$.

How to solve a linear system using a matrix inverse

$$5x + 2y = 3$$

$$4x + 2y = 4$$

$$\begin{bmatrix} 5 & 2 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -2 & \frac{5}{2} \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$

Step 1 – Set up the matrices (AX = B).

- Matrix A will be the coefficients.
- Matrix X will be the variables.
- Matrix B will be constants.

Step 2 – Find the inverse of matrix A ($[A]^{-1}$).

Step 3 – Multiply both sides by $[A]^{-1}$.

Step 4 – Multiply the matrices ($[A]^{-1}[B]$).

Solution (-1, 4)

How to solve a linear system using a matrix inverse

$$x + y + 2z = 3$$
$$2x - y + 3z = -4$$
$$4x - 3y - z = -18$$

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & -1 & 3 \\ 4 & -3 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -4 \\ -18 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} .5 & -.25 & .25 \\ .7 & -.45 & .05 \\ -.1 & .35 & -.15 \end{bmatrix} \begin{bmatrix} 3 \\ -4 \\ -18 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$$

Step 1 – Set up the matrices (AX = B).

- Matrix A will be the coefficients.
- Matrix X will be the variables.
- Matrix B will be constants.

Step 2 – Find the inverse of matrix A ($[A]^{-1}$).

Step 3 – Multiply both sides by $[A]^{-1}$.

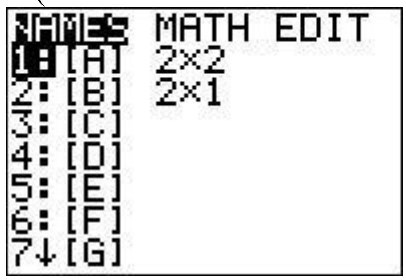
Step 4 – Multiply the matrices.

Solution (-2, 3, 1)

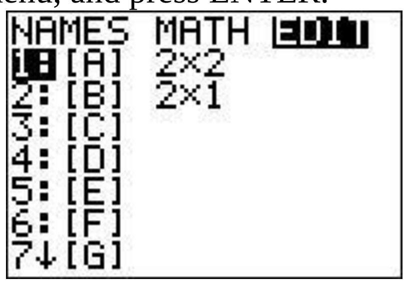
Matrices Using the Graphing Calculator

How to enter matrices into the graphing calculator:

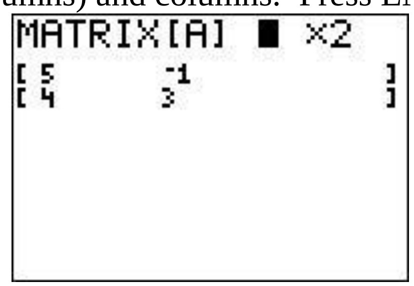
1. Turn on your calculator.
2. Press the "MATRIX" button (You need to use the 2nd button.)



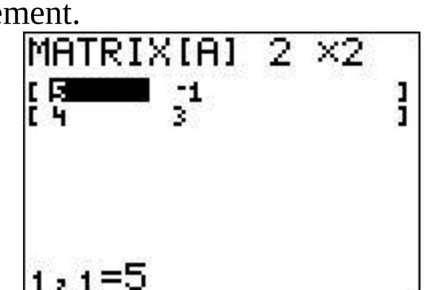
3. Arrow right to the EDIT menu, and press ENTER.



4. Change the matrix dimensions to the correct number of rows (press ENTER to move to the number of columns) and columns. Press ENTER.



5. Your cursor should move to $a_{1,1}$. Enter the elements of your matrix, and press ENTER between each element.

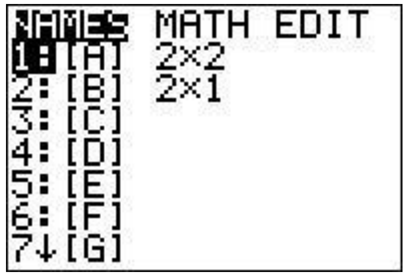


6. When you are done, press the 2nd button then the MODE button. (It will take you to the home screen.)

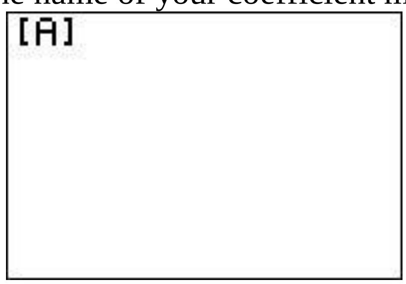
****To solve a linear system of equations, repeat the above steps but enter your constant matrix into a different matrix (for example, [B]).**

How to find the solution of the system:

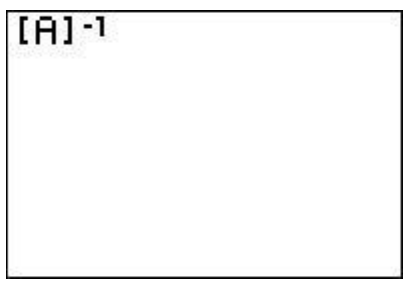
1. From the home screen, press the "MATRIX" button. (You need to use the 2nd button.)



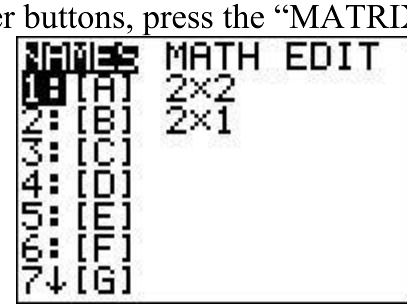
2. Arrow down to highlight the name of your coefficient matrix. Press ENTER.



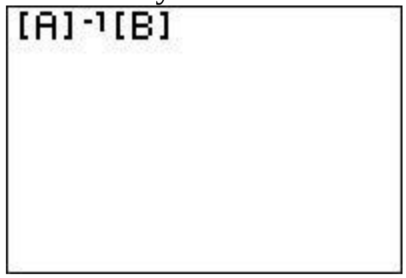
3. You should see your coefficient matrix name on the home screen. Press the x^{-1} button.



4. Without pressing any other buttons, press the "MATRIX" button again.



5. Arrow down to highlight the name of your constant matrix. Press ENTER.

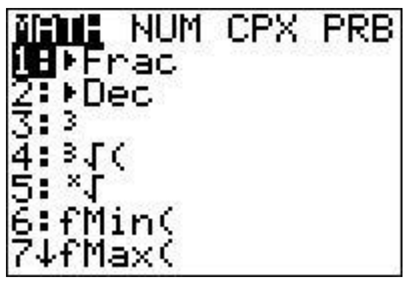


6. Press ENTER to tell the calculator to complete the task.

7. You should see brackets with your solution.

How to find the fraction equivalent:

1. Press the "MATH" button.



2. Press ENTER or 1.

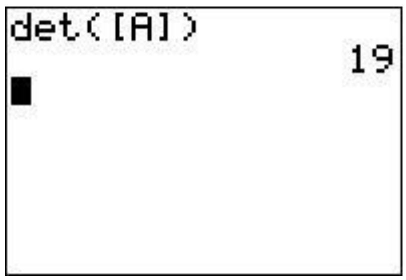
3. Press ENTER.

4. You should see brackets with your solution in fraction form.

How to find the determinant:

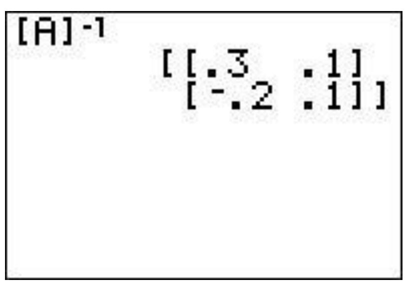
1. Press the "MATRIX" button.
2. Arrow right to the MATH menu, and press ENTER.

3. Without pressing any other buttons, press the "MATRIX" button again.
4. Arrow down to highlight the name of your matrix. Press ENTER.
5. Press ENTER.



How to find the inverse:

1. Press the "MATRIX" button.
2. Arrow down to highlight the name of your matrix. Press ENTER.
3. You should see your coefficient matrix name on the home screen. Press the x^{-1} button.
4. Press ENTER.



5. You should see brackets with your inverse matrix.

Chain Reaction Activity: Matrix Operations
Created by Mary Hinton, Hillwood High School

Materials Needed: Four game cards per group, per round; paper; pencils

Instructions to the teacher for making activity: Copy game cards and separate by cutting them apart.

Optional: paste game cards onto colored index cards, using a different color for each part of each round (card 1: blue; card 2: yellow; card 3: green; card 4: orange) and laminate

Instructions for conducting the activity:
1. Divide class into groups of 4.
2. Students should be seated in rows and given cards in numerical order: Card 1, Card 2, Card 3, Card 4.
3. Calculators are optional
4. Have the answer key on hand during the activity.

Directions to the students:
1. The student who holds card 1 will evaluate the expression.
2. Student 1 will pass his solution to the student who holds card 2, who uses that result to evaluate his/her expression.
3. This process continues until the last student evaluates his/her expression and writes in on a white board or calls it out.
4. The first group to arrive at the correct results wins the round!

*Please contact me if there are any corrections needed for this document or any suggestions☺
Karen.flowers@mnps.org

Retrieved from https://mnpsmath.wikispaces.com/file/view/Chain+Reaction+Algebra+2+Matrices.doc
(MCC9-12.N.VM.7; MCC9-12.N.VM.8)

Answer Key

	X= A + B	**Y = ½ X**	**Z = CY**	**Final answer: -3Z**
Round 1	X = $\begin{bmatrix} 4 \\ 6 \end{bmatrix}$	Y = $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$	Z = $\begin{bmatrix} 28 \end{bmatrix}$	$\begin{bmatrix} -84 \end{bmatrix}$
Round 2	X = $\begin{bmatrix} 4 \\ -6 \end{bmatrix}$	Y = $\begin{bmatrix} 2 \\ -3 \end{bmatrix}$	Z = $\begin{bmatrix} -3 \end{bmatrix}$	$\begin{bmatrix} 9 \end{bmatrix}$
Round 3	X = $\begin{bmatrix} 2 & -4 \\ 2 & 2 \end{bmatrix}$	Y = $\begin{bmatrix} 1 & -2 \\ 1 & 1 \end{bmatrix}$	Z = $\begin{bmatrix} 5 & -1 \\ -5 & 4 \end{bmatrix}$	$\begin{bmatrix} -15 & 3 \\ 15 & -12 \end{bmatrix}$
Round 4	X = $\begin{bmatrix} -2 & 8 \\ 10 & 0 \\ -6 & -8 \end{bmatrix}$	Y = $\begin{bmatrix} -1 & 4 \\ 5 & 0 \\ -3 & -4 \end{bmatrix}$	Z = $\begin{bmatrix} 3 & 8 \\ 56 & 12 \end{bmatrix}$	$\begin{bmatrix} -9 & -24 \\ -168 & -36 \end{bmatrix}$
Round 5	X = $\begin{bmatrix} 10 & -14 \\ -8 & 0 \end{bmatrix}$	Y = $\begin{bmatrix} 5 & -7 \\ -4 & 0 \end{bmatrix}$	Z = $\begin{bmatrix} 2 & -14 \\ 2 & -14 \end{bmatrix}$	$\begin{bmatrix} -6 & 42 \\ -6 & 42 \end{bmatrix}$

Round 1

A = $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ B = $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$ C = $\begin{bmatrix} 5 & 6 \end{bmatrix}$

Round 2

A = $\begin{bmatrix} 0 \\ -1 \end{bmatrix}$ B = $\begin{bmatrix} 4 \\ -5 \end{bmatrix}$ C$\begin{bmatrix} 6 & 5 \end{bmatrix}$

Round 3

A = $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ B = $\begin{bmatrix} 1 & -3 \\ 3 & 1 \end{bmatrix}$ C = $\begin{bmatrix} 2 & 3 \\ -3 & -2 \end{bmatrix}$

Round 4

A = $\begin{bmatrix} 0 & 6 \\ -3 & 4 \\ 5 & -9 \end{bmatrix}$ B = $\begin{bmatrix} -2 & 2 \\ 13 & -4 \\ -11 & 1 \end{bmatrix}$ C = $\begin{bmatrix} 2 & 1 & 0 \\ -3 & 7 & -6 \end{bmatrix}$

Round 5

A = $\begin{bmatrix} 1 & -3 \\ 5 & -7 \end{bmatrix}$ B = $\begin{bmatrix} 9 & -11 \\ -13 & 7 \end{bmatrix}$ C = $\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$

Round 1 Card 1

$A = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $B = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$

X= A + B

Round 1 Card 2

Y = ½ X

Round 1 Card 3

$C = \begin{bmatrix} 5 & 6 \end{bmatrix}$

Z = CY

Round 1 Card 4

Final answer:
-3Z

Round 2 Card 1

$A = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$ $B = \begin{bmatrix} 4 \\ -5 \end{bmatrix}$

X= A + B

Round 2 Card 2

Y = ½ X

Round 2 Card 3

$C = \begin{bmatrix} 6 & 5 \end{bmatrix}$

Z = CY

Round 2 Card 4

Final answer:
-3Z

Round 3 Card 1

$$A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -3 \\ 3 & 1 \end{bmatrix}$$

X= A + B

Round 4 Card 1

$$A = \begin{bmatrix} 0 & 6 \\ -3 & 4 \\ 5 & -9 \end{bmatrix} \quad B = \begin{bmatrix} -2 & 2 \\ 13 & -4 \\ -11 & 1 \end{bmatrix}$$

X= A + B

Round 3 Card 2

Y = ½ X

Round 4 Card 2

Y = ½ X

Round 3 Card 3

$$C = \begin{bmatrix} 2 & 3 \\ -3 & -2 \end{bmatrix}$$

Z = CY

Round 4 Card 3

$$C = \begin{bmatrix} 2 & 1 & 0 \\ -3 & 7 & -6 \end{bmatrix}$$

Z = CY

Round 3 Card 4

Final answer:
-3Z

Round 4 Card 4

Final answer:
-3Z

Round 5 Card 1

$$A = \begin{bmatrix} 1 & -3 \\ 5 & -7 \end{bmatrix} \qquad B = \begin{bmatrix} 9 & -11 \\ -13 & 7 \end{bmatrix}$$

X= A + B

Round 5 Card 2

Y = ½ X

Round 5 Card 3

$$C = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

Z = CY

Round 5 Card 4

Final answer:
-3Z

Cards (alternative form)

Round 1 Card 1

A = $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ B = $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$ C = $\begin{bmatrix} 5 & 6 \end{bmatrix}$

X= A + B

Round 2 Card 1

A = $\begin{bmatrix} 0 \\ -1 \end{bmatrix}$ B = $\begin{bmatrix} 4 \\ -5 \end{bmatrix}$ C$\begin{bmatrix} 6 & 5 \end{bmatrix}$

X= A + B

Round 1 Card 2

A = $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ B = $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$ C = $\begin{bmatrix} 5 & 6 \end{bmatrix}$

Y = ½ X

Round 2 Card 2

A = $\begin{bmatrix} 0 \\ -1 \end{bmatrix}$ B = $\begin{bmatrix} 4 \\ -5 \end{bmatrix}$ C$\begin{bmatrix} 6 & 5 \end{bmatrix}$

Y = ½ X

Round 1 Card 3

A = $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ B = $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$ C = $\begin{bmatrix} 5 & 6 \end{bmatrix}$

Z = CY

Round 2 Card 3

A = $\begin{bmatrix} 0 \\ -1 \end{bmatrix}$ B = $\begin{bmatrix} 4 \\ -5 \end{bmatrix}$ C$\begin{bmatrix} 6 & 5 \end{bmatrix}$

Z = CY

Round 1 Card 4

A = $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ B = $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$ C = $\begin{bmatrix} 5 & 6 \end{bmatrix}$

Final answer:
-3Z

Round 2 Card 4

A = $\begin{bmatrix} 0 \\ -1 \end{bmatrix}$ B = $\begin{bmatrix} 4 \\ -5 \end{bmatrix}$ C$\begin{bmatrix} 6 & 5 \end{bmatrix}$

Final answer:
-3Z

Round 3 Card 1

$$A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -3 \\ 3 & 1 \end{bmatrix}$$
$$C = \begin{bmatrix} 2 & 3 \\ -3 & -2 \end{bmatrix}$$

X= A + B

Round 3 Card 2

$$A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -3 \\ 3 & 1 \end{bmatrix}$$
$$C = \begin{bmatrix} 2 & 3 \\ -3 & -2 \end{bmatrix}$$

Y = ½ X

Round 3 Card 3

$$A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -3 \\ 3 & 1 \end{bmatrix}$$
$$C = \begin{bmatrix} 2 & 3 \\ -3 & -2 \end{bmatrix}$$

Z = CY

Round 3 Card 4

$$A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -3 \\ 3 & 1 \end{bmatrix}$$
$$C = \begin{bmatrix} 2 & 3 \\ -3 & -2 \end{bmatrix}$$

Final answer:
-3Z

Round 4 Card 1

$$A = \begin{bmatrix} 0 & 6 \\ -3 & 4 \\ 5 & -9 \end{bmatrix} \quad B = \begin{bmatrix} -2 & 2 \\ 13 & -4 \\ -11 & 1 \end{bmatrix}$$
$$C = \begin{bmatrix} 2 & 1 & 0 \\ -3 & 7 & -6 \end{bmatrix}$$

X= A + B

Round 4 Card 2

$$A = \begin{bmatrix} 0 & 6 \\ -3 & 4 \\ 5 & -9 \end{bmatrix} \quad B = \begin{bmatrix} -2 & 2 \\ 13 & -4 \\ -11 & 1 \end{bmatrix}$$
$$C = \begin{bmatrix} 2 & 1 & 0 \\ -3 & 7 & -6 \end{bmatrix}$$

Y = ½ X

Round 4 Card 3

$$A = \begin{bmatrix} 0 & 6 \\ -3 & 4 \\ 5 & -9 \end{bmatrix} \quad B = \begin{bmatrix} -2 & 2 \\ 13 & -4 \\ -11 & 1 \end{bmatrix}$$
$$C = \begin{bmatrix} 2 & 1 & 0 \\ -3 & 7 & -6 \end{bmatrix}$$

Z = CY

Round 4 Card 4

$$A = \begin{bmatrix} 0 & 6 \\ -3 & 4 \\ 5 & -9 \end{bmatrix} \quad B = \begin{bmatrix} -2 & 2 \\ 13 & -4 \\ -11 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 2 & 1 & 0 \\ -3 & 7 & -6 \end{bmatrix}$$

Final answer:
-3Z

Round 5 Card 1

$$A = \begin{bmatrix} 1 & -3 \\ 5 & -7 \end{bmatrix}$$

$$B = \begin{bmatrix} 9 & -11 \\ -13 & 7 \end{bmatrix}$$

$$C = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

X= A + B

Round 5 Card 2

$$A = \begin{bmatrix} 1 & -3 \\ 5 & -7 \end{bmatrix}$$

$$B = \begin{bmatrix} 9 & -11 \\ -13 & 7 \end{bmatrix}$$

$$C = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

Y = ½ X

Round 5 Card 3

$$A = \begin{bmatrix} 1 & -3 \\ 5 & -7 \end{bmatrix}$$

$$B = \begin{bmatrix} 9 & -11 \\ -13 & 7 \end{bmatrix}$$

$$C = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

Z = CY

Round 5 Card 4

$$A = \begin{bmatrix} 1 & -3 \\ 5 & -7 \end{bmatrix}$$

$$B = \begin{bmatrix} 9 & -11 \\ -13 & 7 \end{bmatrix}$$

$$C = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$$

Final answer:
-3Z