

B.U.G. Newsletter



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IN THIS ISSUE

As one semester comes to an end...

by Jennifer L. Brown

It is hard to believe that December is upon us. Gee! How he time flies. My once newborn son is now six months old. As one semester comes to an end, we must begin to think about the upcoming units. In Coordinate Algebra, Unit 4 focuses on describing data. Yes, as we have discussed in earlier newsletters, data analysis is my favorite content. For the activities this month, I have included two linear regression lessons. One of them is "The Wandering Point". (Note: The accompanying PowerPoint is available on my website.) This lesson teaches the students about residuals using the clearboards from the summer workshop. The second lesson involves hoola hoops to create a linear equation that can be used for prediction. The students will enjoy the data collection procedures.

In Analytic Geometry, Unit 5 focuses on the quadratic function. I included a graphic organizer for teaching simple linear and quadratic system of equations – both graphically and algebraically. In Advanced Algebra, Unit 4 focus on exponentials and

logarithms. Within that unit, the students will need to apply the concept of an inverse relationship. I am including several activities from my classroom that introduces that concept. These activities can be used in Unit 6, too. First, introducing the inverse relationship using a table of values and graphing with linear and exponential functions. This activity will be a great activity for your clearboards, too. (Note: The PowerPoint file is posted on my website.) Second, there is a graphic organizer with what an inverse is, how to find an inverse, and how to verify an inverse. If you like foldables, I included the foldable option, too. In addition, the students will need to revisit operations with polynomials from Unit 2. I included a graphic organizer to review that content. The Word files and/or PowerPoint files for these activities are available for download from my website if you would like to edit, add, or delete anything. If you have any questions, please feel free to contact me.

Dr. Brown ☺

Prior Knowledge	2
The Wandering Point (Student).....	3
Hoola Hoop Challenge	4
Solving Linear and Quadratic Systems	7
Inverse Graphic Organizer (Student) .	8
Inverse Graphic Organizer (Teacher) .	9
Inverse Functions Foldable	10
Operations with Functions Review Graphic Organizer (Student)	12
Operations with Functions Review Graphic Organizer (Teacher).....	13



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Prior Knowledge

Unit 4 in Coordinate Algebra

- Know how to compute the mean, median, interquartile range, and mean standard deviation by hand in simple cases and using technology with larger data sets.
- Find the lower extreme (minimum), upper extreme (maximum), and quartiles.
- Create a graphical representation of a data set.
- Present data in a frequency table.
- Plot data on a coordinate grid and graph linear functions.

- Recognize characteristics of linear and exponential functions.
- Write an equation of a line given two points.
- Graph data in a scatter plot and determine a trend.
- Determine the slope of a line from any representation.
- Identify the y intercept from any representation.
- Be able to use graphing technology.
- Understand the meaning of correlation.

Unit 5 in Analytic Geometry

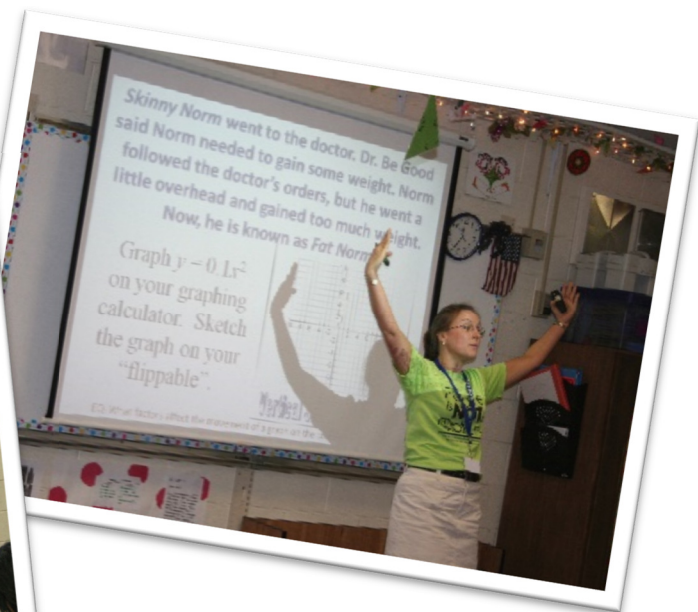
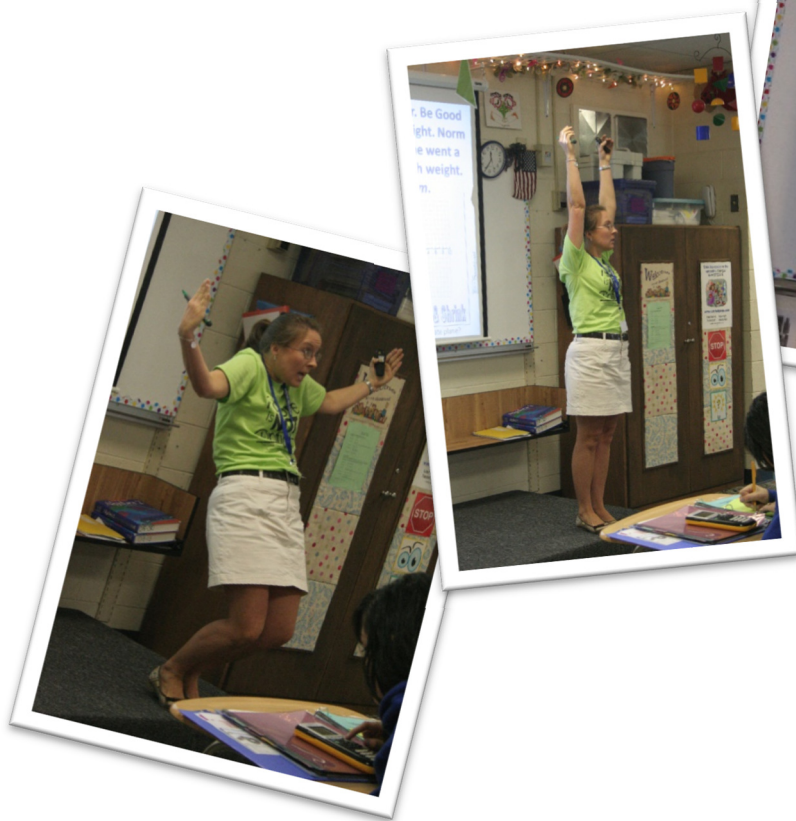
- Use function notation.
- Put data into tables.
- Graph data from tables.

- Solve one variable linear equations.
- Determine domain of a problem situation.
- Solve for any variable in a multi-variable equation.
- Recognize slope of a linear function as a rate of change.
- Graph linear functions.
- Use of complex numbers.
- Graph inequalities.

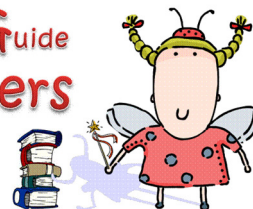
Unit 4 in Advanced Algebra

- Apply the concept of a function.
- Apply various representations of functions.
- Understand exponential functions and the characteristics of their graphs.
- Solve linear equations using algebra and graphing methods.
- Familiar with graphing technology.
- Use patterns to write a function to model a situation.

Dr. Brown teaching a lesson on transformations with quadratic functions to high school students.



**Bell's Useful Guide
for Teachers**



THE WANDERING POINT

Purpose

- Determine the residuals of a linear model using informal and formal methods.

Materials

- Clearboard (1 per student)
- Graph template
- 4 wikki sticks
- Graphing calculator

Procedures

1. Use your wikki stick to “eyeball” a line of best fit.
2. Find \hat{y} (predicted y value) for each x value.
3. Find the informal residual for each x value.
4. Find $y - \hat{y}$ (exact residual) for each x value.

Questions

1. For graph #1, which point(s) have a small residual? large residual?

2. For graph #2, which point(s) have a small residual? large residual?

3. For graph #3, which point(s) have a small residual? large residual?

4. For graph #4, which point(s) have a small residual? large residual?

Graphing:

1. On a separate piece of graph paper, plot the collected data. Be sure to label your axes with variables and units.
2. Draw a line through your data by moving a straightedge until you believe that you have the line of best fit. It should pass as close to the data points as possible with some points above and others below.

Analysis:

1. Find an equation in slope-intercept form ($y = mx + b$) using two points from your data.

$y =$ _____

2. What is the y-intercept of your line (units)? $b =$ _____

3. What does the y-intercept mean in real-life terms in this experiment? Explain your answer.

4. What is the slope of your line (units)? $m =$ _____

5. What does the slope mean in real-life terms in this experiment? Explain your answer.

6. Using your equation in number 1, predict how long it would take for 100 people to complete the hoola-hoop pass. (Show your work!)

7. Using your equation in number 1, predict how many people could have been involved in the activity if the hoola-hoop pass took one hour. (Show your work! Do not forget to convert the time!)

8. Are your answers to 6 and 7 reasonable? Explain your rationale.

9. Plot your data using the graphing calculator (see directions below) and conduct a linear regression.

$y =$ _____

10. Compare the linear regression equation to the equation you found in number 1. Which one is a better fit? Explain your rationale.

To plot points on the TI-83:

To enter the data
STAT
1: EDIT <ENTER>
In L₁, enter the length list of the
MATCHING data
In L₂, enter the corresponding average
times

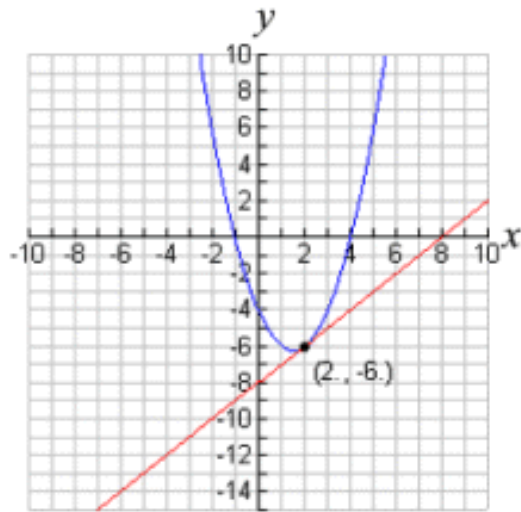
To view the scatterplot
2nd Y=
1: Plot1...Off <ENTER>
On
Type: <first one – scatter>
Xlist: L₁
Ylist: L₂
Mark: .
WINDOW
Enter the appropriate window from your
graph on previous page
GRAPH

To conduct the linear regression:

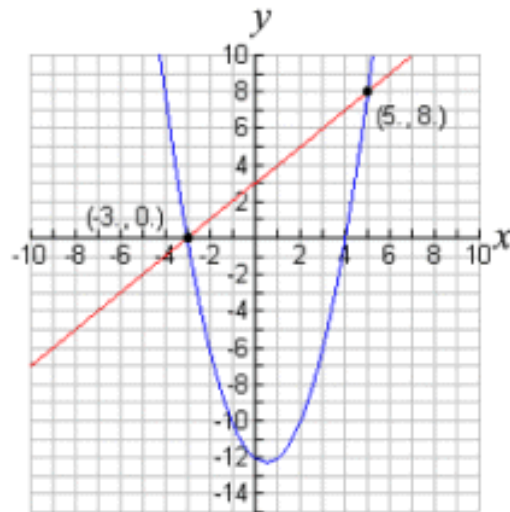
2ND MODE
CLEAR
STAT
CALC
4:LinReg (ax+b)
2nd 1
,
2nd 2
,
VARS
Y-VARS
1:FUNCTION <enter>
1:Y1 <enter>

****This procedure will find the line of best fit and paste it into your Y= screen.**

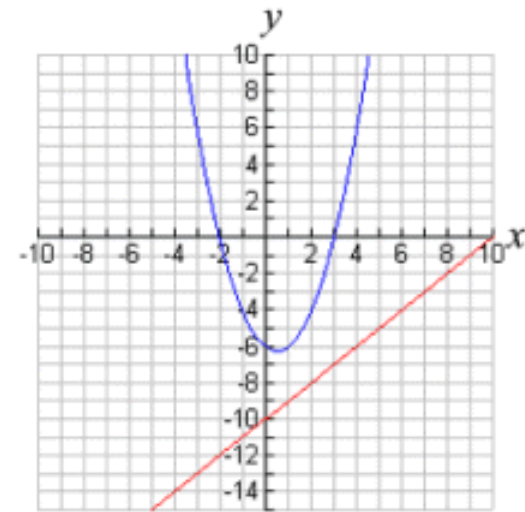
Systems of Linear-Quadratic Equations



One Solution



Two Solutions



No Solution

Steps for Solving Systems of Linear-Quadratic Equations (Algebraically):

1. Set the functions equal to each other.
2. Solve for x.
3. Plug in the value(s) for x & solve for y.

$$\begin{aligned} f(x) &= x - 8 \\ g(x) &= x^2 - 3x - 4 \end{aligned}$$

$$\begin{array}{r} x - 8 = x^2 - 3x - 4 \\ -x \qquad \qquad -x \\ \hline -8 = x^2 - 4x - 4 \\ +8 \qquad \qquad +8 \\ \hline 0 = x^2 - 4x + 4 \\ 0 = (x - 2)(x - 2) \\ x = 2 \end{array}$$

$$\begin{aligned} (2) - 8 &= -6 \\ \mathbf{(2, -6)} \end{aligned}$$

$$\begin{aligned} f(x) &= x + 3 \\ g(x) &= x^2 - x - 12 \end{aligned}$$

$$\begin{array}{r} x + 3 = x^2 - x - 12 \\ -x \qquad \qquad -x \\ \hline 3 = x^2 - 2x - 12 \\ -3 \qquad \qquad -3 \\ \hline 0 = x^2 - 2x - 15 \\ 0 = (x - 5)(x + 3) \\ x = 5 \quad x = -3 \end{array}$$

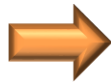
$$\begin{aligned} (5) + 3 &= 8 & (-3) + 3 &= 8 \\ \mathbf{(5, 8)} & & \mathbf{(5, 0)} & \end{aligned}$$

$$\begin{aligned} f(x) &= x - 10 \\ g(x) &= x^2 - x - 6 \end{aligned}$$

$$\begin{array}{r} x - 10 = x^2 - x - 6 \\ -x \qquad \qquad -x \\ \hline -10 = x^2 - 2x - 6 \\ +10 \qquad \qquad +10 \\ \hline 0 = x^2 - 2x + 4 \end{array}$$

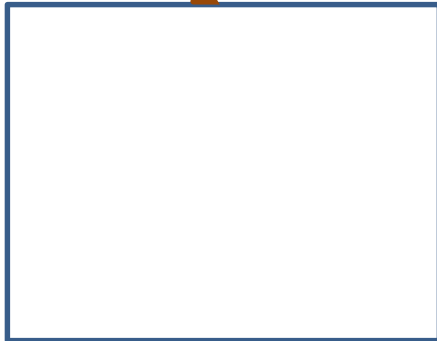
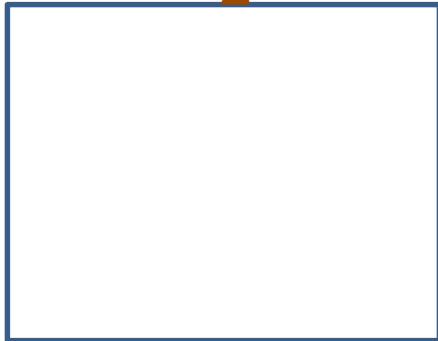
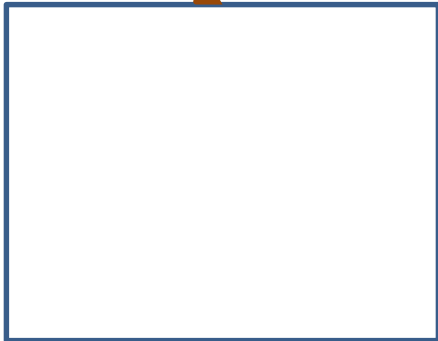
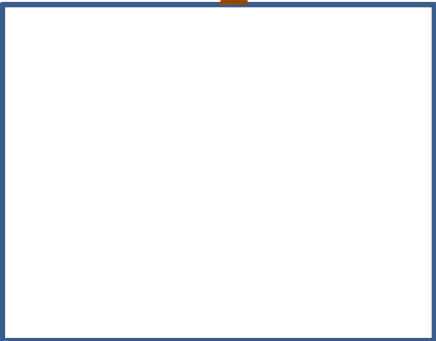
no real solution

What is an inverse?



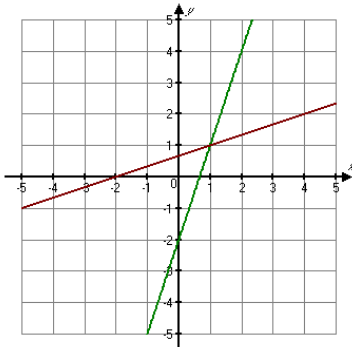
How to FIND an Inverse

How to VERIFY an Inverse



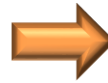
x	$f(x)$	x	$f^{-1}(x)$
-1		5	
0		-2	
2		4	
3		7	

$f(x) = 3x - 2$



$f(f^{-1}(x)) = 3\left(\frac{x+2}{3}\right) - 2$

What is an inverse?



Opposite

Think left & right or up & down!



How to FIND an Inverse

How to VERIFY an Inverse

Coordinates

Switch the domain & range in each coordinate.

Equation

1. Switch the x & y.
2. Solve for y.

Graphing

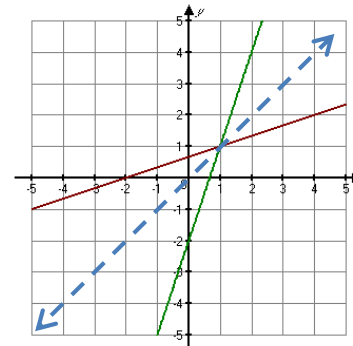
Do the graphs reflect across $y = x$?

Composition of Functions

Does $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$?

x	f(x)	x	f ⁻¹ (x)
-1	5	5	-1
0	-2	-2	0
2	4	4	2
3	7	7	3

$$\begin{aligned}
 f(x) &= 3x - 2 \\
 x &= 3y - 2 \\
 +2 & \quad +2 \\
 \hline
 x+2 &= 3y \\
 3 & \quad \cancel{3} \\
 f^{-1}(x) &= \frac{x+2}{3}
 \end{aligned}$$



$$\begin{aligned}
 f(f^{-1}(x)) &= \cancel{3} \left(\frac{x+2}{\cancel{3}} \right) - 2 \\
 &= x + \cancel{2} - \cancel{2} \\
 &= x \\
 f^{-1}(f(x)) &= \frac{(3x - \cancel{2}) + \cancel{2}}{3} \\
 &= \frac{\cancel{3}x}{\cancel{3}} \\
 &= x
 \end{aligned}$$

STEPS

STEPS

STEPS

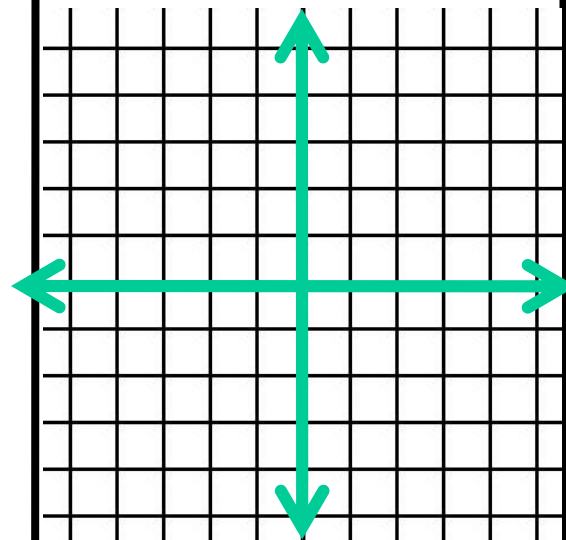
STEPS

EXAMPLE

EXAMPLE

EXAMPLE

EXAMPLE



Inverse Functions

$$f^{-1}(x)$$

Verify an inverse

of functions

Composition

Reflection
across
 $y = x$

Find an inverse

the x & y

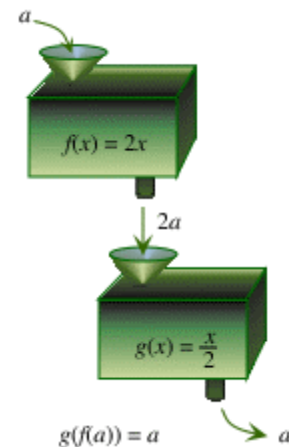
Switch

Switch the
domain
& range

Operations of Functions

$$f(x) = x^2 + 2x + 1$$

$$g(x) = 3x + 2$$



**Sum
(Add)**

$$(f + g)(x) = f(x) + g(x)$$

**Difference
(Subtract)**

$$(f - g)(x) = f(x) - g(x)$$

**Product
(Multiply)**

$$(f \cdot g)(x) = f(x) \cdot g(x)$$

**Quotient
(Divide)**

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

$g(x) \neq 0$

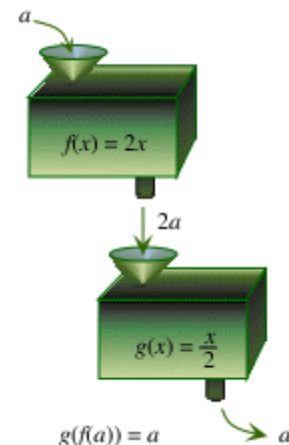
**Composition
of Functions
(Substitute)**

$$(g \circ f)(x) = g(f(x))$$

Operations of Functions

$$f(x) = x^2 + 2x + 1$$

$$g(x) = 3x + 2$$



Sum (Add)

$$(f + g)(x) = f(x) + g(x)$$

Difference (Subtract)

$$(f - g)(x) = f(x) - g(x)$$

Product (Multiply)

$$(f \cdot g)(x) = f(x) \cdot g(x)$$

Quotient (Divide)

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

$g(x) \neq 0$

Composition of Functions (Substitute)

$$(g \circ f)(x) = g(f(x))$$

$$\begin{aligned} f(x) + g(x) &= x^2 + 2x + 1 \\ &\quad + 3x + 2 \\ &= x^2 + 5x + 3 \end{aligned}$$

$$\begin{aligned} f(x) - g(x) &= x^2 + 2x + 1 \\ &\quad + -3x + 2 \\ &= x^2 - x - 1 \end{aligned}$$

$$f(x) \cdot g(x) = (x^2 + 2x + 1)(3x + 2)$$

$$\begin{array}{r} x^2 \quad +2x \quad +1 \\ 3x \end{array}$$

	$3x^3$	$+6x^2$	$+3x$
$+2$	$+2x^2$	$+4x$	$+2$

$$= 3x^3 + 8x^2 + 7x + 2$$

$$f(x) \div g(x) = \frac{(x^2 + 2x + 1)}{(3x + 2)}$$

$$\begin{aligned} g(f(x)) &= g(x^2 + 2x + 1) \\ &= 3(x^2 + 2x + 1) + 2 \\ &= 3x^2 + 6x + 3 + 2 \\ &= 3x^2 + 6x + 5 \end{aligned}$$